

5.3 Applications of Polynomials

In this section we investigate real-world applications of polynomial functions.

You Try It!

EXAMPLE 1. The average price of a gallon of gas at the beginning of each month for the period starting in November 2010 and ending in May 2011 are given in the margin. The data is plotted in Figure 5.18 and fitted with the following third degree polynomial, where t is the number of months that have passed since October of 2010.

$$p(t) = -0.0080556t^3 + 0.11881t^2 - 0.30671t + 3.36 \quad (5.2)$$

Use the graph and then the polynomial to estimate the price of a gallon of gas in California in February 2011.

| Month | Price |
|-------|-------|
| Nov. | 3.14 |
| Dec. | 3.21 |
| Jan | 3.31 |
| Mar. | 3.87 |
| Apr. | 4.06 |
| May | 4.26 |

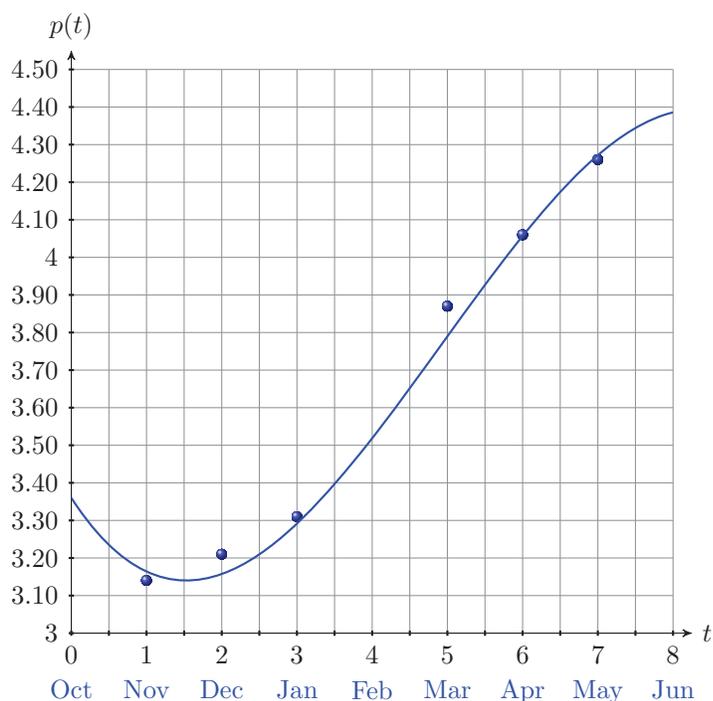


Figure 5.18: Fitting gas price versus month with a cubic polynomial.

Solution: Locate February ($t = 4$) on the horizontal axis. From there, draw a vertical arrow up to the graph, and from that point of intersection, a second horizontal arrow over to the vertical axis (see Figure 5.19). It would appear that the price per gallon in February was approximately \$3.51.

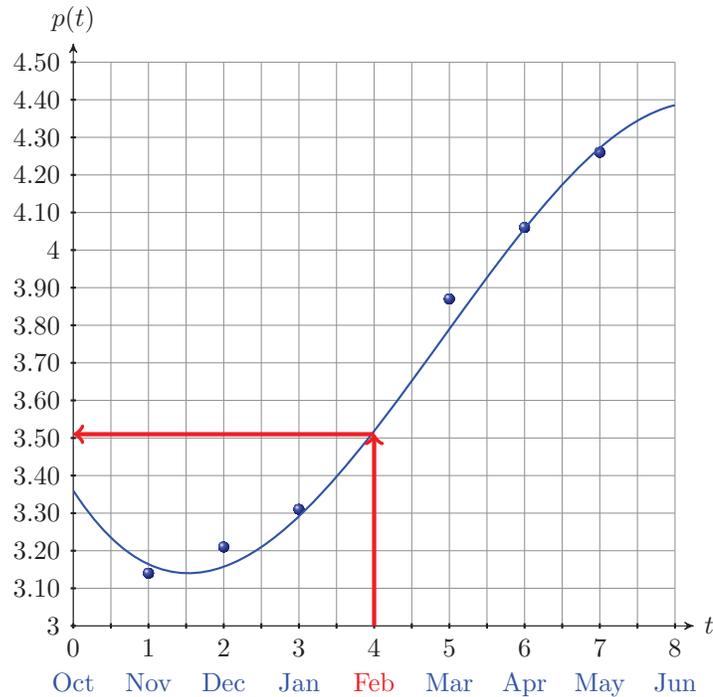


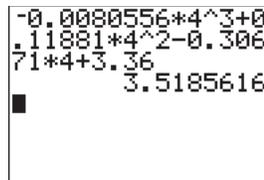
Figure 5.19: Approximating price of gas during February.

Next, we'll use the fitted third degree polynomial to approximate the price per gallon for the month of February, 2011. Start with the function defined by [equation 5.2](#) and substitute 4 for t .

$$p(t) = -0.0080556t^3 + 0.11881t^2 - 0.30671t + 3.36$$

$$p(4) = -0.0080556(4)^3 + 0.11881(4)^2 - 0.30671(4) + 3.36$$

Use the calculator to evaluate $p(4)$ (see [Figure 5.20](#)). Rounding to the nearest

Figure 5.20: Evaluating $p(4)$.

penny, the price in February was \$3.52 per gallon.



You Try It!

EXAMPLE 2. If a projectile is fired into the air, its height above ground at any time is given by the formula

$$y = y_0 + v_0t - \frac{1}{2}gt^2, \quad (5.3)$$

where

- y = height above ground at time t ,
- y_0 = initial height above ground at time $t = 0$,
- v_0 = initial velocity at time $t = 0$,
- g = acceleration due to gravity,
- t = time passed since projectile's firing.

If a projectile is launched with an initial velocity of 60 meters per second from a rooftop 12 meters above ground level, at what time will the projectile first reach a height of 150 meters?

If a projectile is launched with an initial velocity of 100 meters per second (100 m/s) from a rooftop 8 meters (8 m) above ground level, at what time will the projectile first reach a height of 400 meters (400 m)? *Note: Near the earth's surface, the acceleration due to gravity is approximately 9.8 meters per second per second (9.8 (m/s)/s or 9.8 m/s²).*

Solution: We're given the initial height is $y_0 = 8$ m, the initial velocity is $v_0 = 100$ m/s, and the acceleration due to gravity is $g = 9.8$ m/s². Substitute these values in [equation 5.3](#), then simplify to produce the following result:

$$\begin{aligned} y &= y_0 + v_0t - \frac{1}{2}gt^2 \\ y &= 8 + 100t - \frac{1}{2}(9.8)t^2 \\ y &= 8 + 100t - 4.9t^2 \end{aligned}$$

Enter $y = 8 + 100t - 4.9t^2$ as **Y1=8+100*X-4.9*X^2** in the Y= menu (see the first image in [Figure 5.21](#)). After some experimentation, we settled on the WINDOW parameters shown in the second image in [Figure 5.21](#). Push the GRAPH button to produce the graph of $y = 8 + 100t - 4.9t^2$ shown in the third image [Figure 5.21](#).

In this example, the horizontal axis is actually the t -axis. So when we set **Xmin** and **Xmax**, we're actually setting bounds on the t -axis.

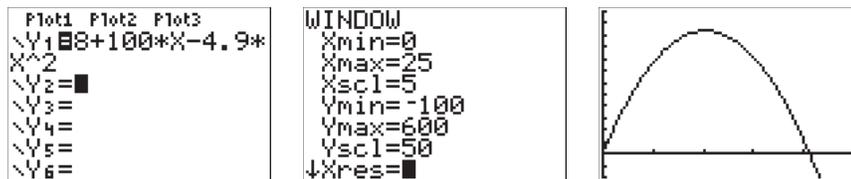


Figure 5.21: Sketching the graph of $y = 8 + 100t - 4.9t^2$.

To find when the projectile reaches a height of 400 meters (400 m), substitute 400 for y to obtain:

$$400 = 8 + 100t - 4.9t^2 \quad (5.4)$$

Enter the left-hand side of Equation 5.4 into **Y2** in the Y= menu, as shown in the first image in Figure 5.22. Push the GRAPH button to produce the result shown in the second image in Figure 5.22. Note that there are two points of intersection, which makes sense as the projectile hits 400 meters on the way up and 400 meters on the way down.

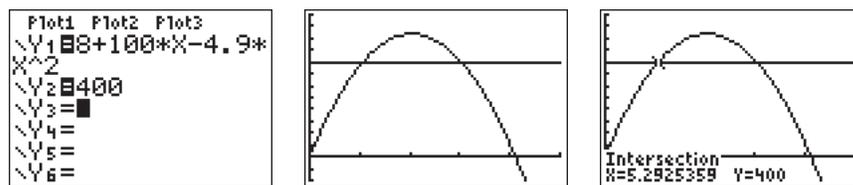


Figure 5.22: Determining when the object first reaches 400 meters.

To find the first point of intersection, select **5:intersect** from the CALC menu. Press ENTER in response to “First curve,” then press ENTER again in response to “Second curve.” For your guess, use the arrow keys to move the cursor closer to the first point of intersection than the second. At this point, press ENTER in response to “Guess.” The result is shown in the third image in Figure 5.22. The projectile first reaches a height of 400 meters at approximately 5.2925359 seconds after launch.

The parabola shown in Figure 5.23 is not the actual flight path of the projectile. The graph only predicts the height of the projectile as a function of time.

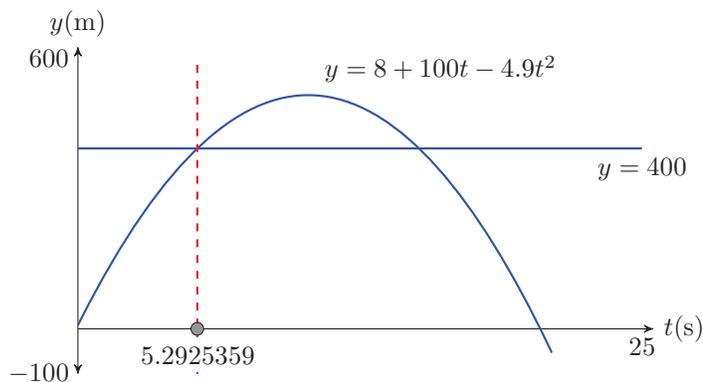


Figure 5.23: Reporting your graphical solution on your homework.

Reporting the solution on your homework: Duplicate the image in your calculator’s viewing window on your homework page. Use a ruler to draw all lines, but freehand any curves.

- Label the horizontal and vertical axes with t and y , respectively (see Figure 5.23). Include the units (seconds (s) and meters (m)).
- Place your WINDOW parameters at the end of each axis (see Figure 5.23). Include the units (seconds (s) and meters (m)).
- Label each graph with its equation (see Figure 5.23).
- Draw a dashed vertical line through the first point of intersection. Shade and label the point (with its t -value) where the dashed vertical line crosses the t -axis. This is the first solution of the equation $400 = 8 + 100t - 4.9t^2$ (see Figure 5.23).

The phrase “shade and label the point” means fill in the point on the t -axis, then write the t -value of the point just below the shaded point.

Rounding to the nearest tenth of a second, it takes the projectile approximately $t \approx 5.3$ seconds to first reach a height of 400 meters.

Answer:
 ≈ 3.0693987 seconds

Zeros and x -intercepts of a Function

Recall that $f(x)$ and y are interchangeable. Therefore, if we are asked to find where a function is equal to zero, then we need to find the points on the graph of the function that have a y -value equal to zero (see Figure 5.24).

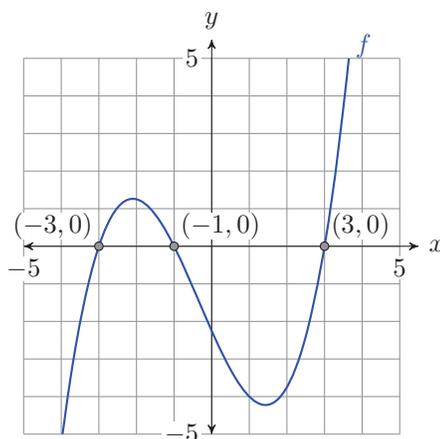


Figure 5.24: Locating the zeros of a function.

Zeros and x -intercepts. The points where the graph of f crosses the x -axis are called the x -intercepts of the graph of f . The x -value of each x -intercept is called a *zero* of the function f .

The graph of f crosses the x -axis in Figure 5.24 at $(-3, 0)$, $(-1, 0)$, and $(3, 0)$. Therefore:

- The x -intercepts of f are: $(-3, 0)$, $(-1, 0)$, and $(3, 0)$
- The zeros of f are: -3 , -1 , and 3

Key idea. A function is zero where its graph crosses the x -axis.

You Try It!

Find the zero(s) of the function $f(x) = 2.6x - 9.62$.

EXAMPLE 3. Find the zero(s) of the function $f(x) = 1.5x + 5.25$.

Algebraic solution: Remember, $f(x) = 1.5x + 5.25$ and $y = 1.5x + 5.25$ are equivalent. We're looking for the value of x that makes $y = 0$ or $f(x) = 0$. So, we'll start with $f(x) = 0$, then replace $f(x)$ with $1.5x + 5.25$.

$$\begin{array}{ll} f(x) = 0 & \text{We want the value of } x \text{ that makes} \\ & \text{the function equal to zero.} \\ 1.5x + 5.25 = 0 & \text{Replace } f(x) \text{ with } 1.5x + 5.25. \end{array}$$

Now we solve for x .

$$\begin{array}{ll} 1.5x = -5.25 & \text{Subtract } 5.25 \text{ from both sides.} \\ x = \frac{-5.25}{1.5} & \text{Divide both sides by } 1.5. \\ x = -3.5 & \text{Divide: } -5.25/1.5 = -3.5 \end{array}$$

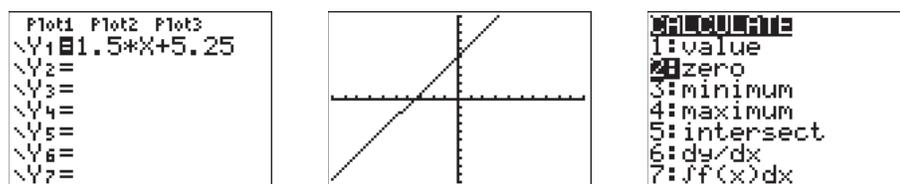
Check. Substitute -3.5 for x in the function $f(x) = 1.5x + 5.25$.

$$\begin{array}{ll} f(x) = 1.5x + 5.25 & \text{The original function.} \\ f(-3.5) = 1.5(-3.5) + 5.25 & \text{Substitute } -3.5 \text{ for } x. \\ f(-3.5) = -5.25 + 5.25 & \text{Multiply: } 1.5(-3.5) = -5.25. \\ f(-3.5) = 0 & \text{Add.} \end{array}$$

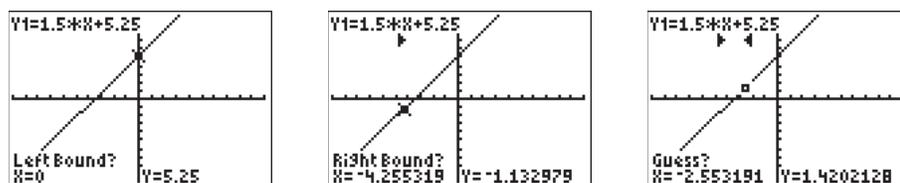
Note that -3.5 , when substituted for x , makes the function $f(x) = 1.5x + 5.25$ equal to zero. This is why -3.5 is called a *zero* of the function.

Graphing calculator solution. We should be able to find the zero by sketching the graph of f and noting where it crosses the x -axis. Start by loading the function $f(x) = 1.5x + 5.25$ into **Y1** in the **Y=** menu (see the first image in Figure 5.25).

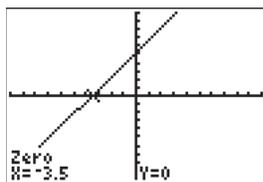
Select **6:ZStandard** from the **ZOOM** menu to produce the graph of f (see the second image in Figure 5.25). Press **2ND CALC** to open the **CALCULATE** menu (see the third image in Figure 5.25). To find the zero of the function f :

Figure 5.25: Finding the zero of $f(x) = 1.5x + 5.25$.

1. Select **2:zero** from the CALCULATE menu. The calculator responds by asking for a “Left Bound?” (see the first image in Figure 5.26). Use the left arrow button to move the cursor so that it lies to the left of the x -intercept of f and press ENTER.
2. The calculator responds by asking for a “Right Bound?” (see the second image in Figure 5.26). Use the right arrow button to move the cursor so that it lies to the right of the x -intercept of f and press ENTER.
3. The calculator responds by asking for a “Guess?” (see the third image in Figure 5.26). As long as your cursor lies between the left- and right-bound marks at the top of the screen (see the third image in Figure 5.26), you have a valid guess. Since the cursor already lies between the left- and right-boundaries, simply press ENTER to use the current position of the cursor as your guess.

Figure 5.26: Using **2:zero** from the CALCULATE menu.

The calculator responds by approximating the zero of the function as shown in Figure 5.27.

Figure 5.27: -3.5 is a zero of f .

Note that the approximation found using the calculator agrees nicely with the zero found using the algebraic technique.

Answer: 3.7

You Try It!

If a projectile is launched with an initial velocity of 60 meters per second from a rooftop 12 meters above ground level, at what time will the projectile return to ground level?

EXAMPLE 4. How long will it take the projectile in [Example 2](#) to return to ground level?

Solution: In [Example 2](#), the height of the projectile above the ground as a function of time is given by the equation

$$y = 8 + 100t - 4.9t^2.$$

When the projectile returns to the ground, its height above ground will be zero meters. To find the time that this happens, substitute $y = 0$ in the last equation and solve for t .

$$0 = 8 + 100t - 4.9t^2$$

Enter the equation $y = 8 + 100t - 4.9t^2$ into **Y1** in the **Y=** menu of your calculator (see the first image in [Figure 5.28](#)), then set the **WINDOW** parameters shown in the second image in [Figure 5.28](#). Push the **GRAPH** button to produce the graph of the function shown in the third image in [Figure 5.28](#).

In this example, the horizontal axis is actually the t -axis. So when we set **Xmin** and **Xmax**, we're actually setting bounds on the t -axis.

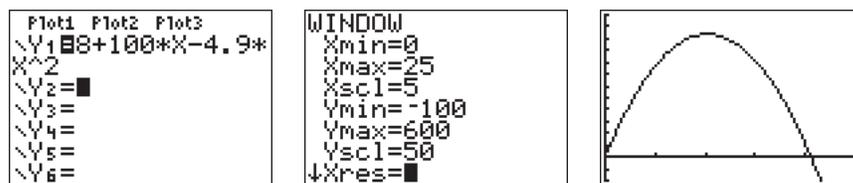


Figure 5.28: Sketching the graph of $y = 8 + 100t - 4.9t^2$.

To find the time when the projectile returns to ground level, we need to find where the graph of $y = 8 + 100t - 4.9t^2$ crosses the horizontal axis (in this case the t -axis). Select **2:zero** from the **CALC** menu. Use the arrow keys to move the cursor slightly to the left of the t -intercept, then press **ENTER** in response to “Left bound.” Move your cursor slightly to the right of the t -intercept, then press **ENTER** in response to “Right bound.” Leave your cursor where it is and press **ENTER** in response to “Guess.” The result is shown in [Figure 5.29](#).

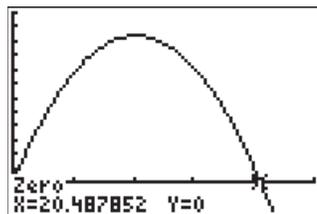


Figure 5.29: Finding the time when the projectile hits the ground.

- Label the horizontal and vertical axes with t and y , respectively (see Figure 5.30). Include the units (seconds (s) and meters (m)).
- Place your WINDOW parameters at the end of each axis (see Figure 5.30).
- Label the graph with its equation (see Figure 5.30).
- Draw a dashed vertical line through the t -intercept. Shade and label the t -value of the point where the dashed vertical line crosses the t -axis. This is the solution of the equation $0 = 8 + 100t - 4.9t^2$ (see Figure 5.30).

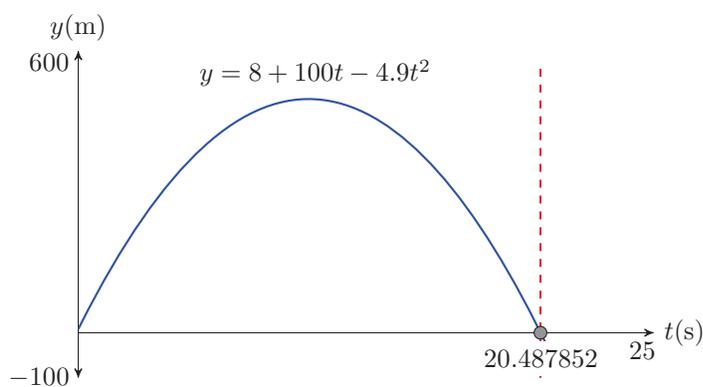


Figure 5.30: Reporting your graphical solution on your homework.

Rounding to the nearest tenth of a second, it takes the projectile approximately $t \approx 20.5$ seconds to hit the ground.

Answer:
 ≈ 12.441734 seconds

□

🐞 🐞 🐞 **Exercises** 🐞 🐞 🐞

1. A firm collects data on the amount it spends on advertising and the resulting revenue collected by the firm. Both pieces of data are in thousands of dollars.

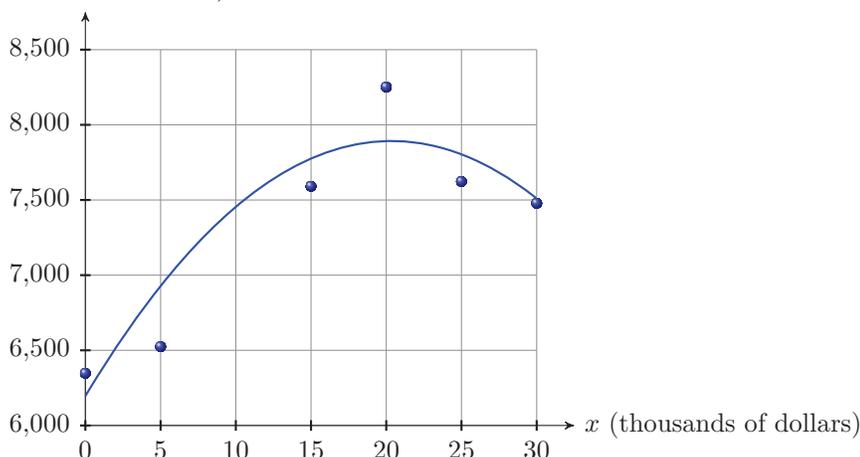
| | | | | | | |
|-------------------------|------|------|------|------|------|------|
| x (advertising costs) | 0 | 5 | 15 | 20 | 25 | 30 |
| R (revenue) | 6347 | 6524 | 7591 | 8251 | 7623 | 7478 |

The data is plotted then fitted with the following second degree polynomial, where x is the amount invested in thousands of dollars and $R(x)$ is the amount of revenue earned by the firm (also in thousands of dollars).

$$R(x) = -4.1x^2 + 166.8x + 6196$$

Use the graph and then the polynomial to estimate the firm's revenue when the firm invested \$10,000 in advertising.

R (thousands of dollars)



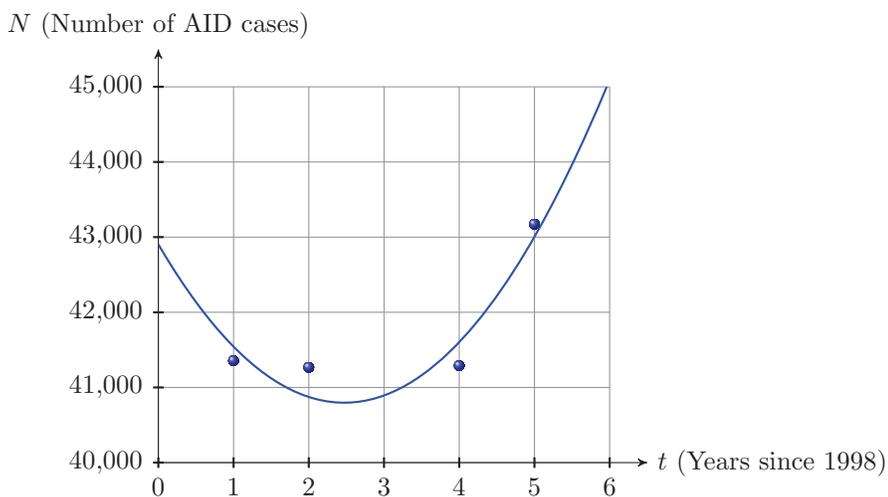
2. The table below lists the estimated number of aids cases in the United States for the years 1999-2003.

| | | | | |
|------------|--------|--------|--------|--------|
| Year | 1999 | 2000 | 2002 | 2003 |
| AIDS Cases | 41,356 | 41,267 | 41,289 | 43,171 |

The data is plotted then fitted with the following second degree polynomial, where t is the number of years that have passed since 1998 and $N(t)$ is the number of aids case reported t years after 1998.

$$N(t) = 345.14t^2 - 1705.7t + 42904$$

Use the graph and then the polynomial to estimate the number of AIDS cases in the year 2001.



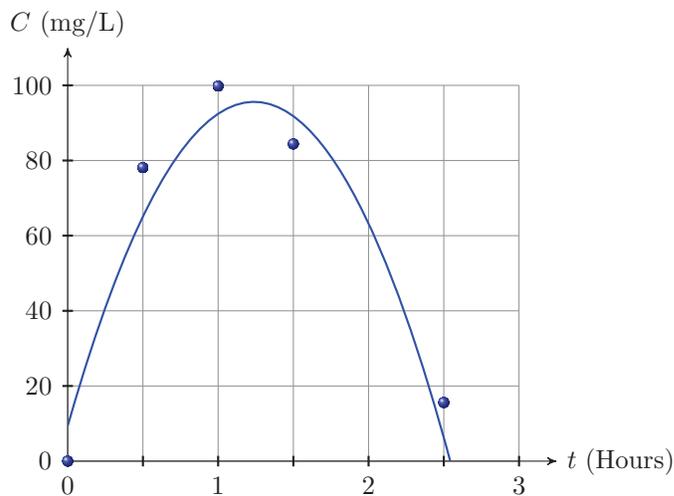
3. The following table records the concentration (in milligrams per liter) of medication in a patient's blood after indicated times have passed.

| Time (Hours) | 0 | 0.5 | 1 | 1.5 | 2.5 |
|----------------------|---|------|------|------|------|
| Concentration (mg/L) | 0 | 78.1 | 99.8 | 84.4 | 15.6 |

The data is plotted then fitted with the following second degree polynomial, where t is the number of hours that have passed since taking the medication and $C(t)$ is the concentration (in milligrams per liter) of the medication in the patient's blood after t hours have passed.

$$C(t) = -56.214t^2 + 139.31t + 9.35$$

Use the graph and then the polynomial to estimate the the concentration of medication in the patient's blood 2 hours after taking the medication.



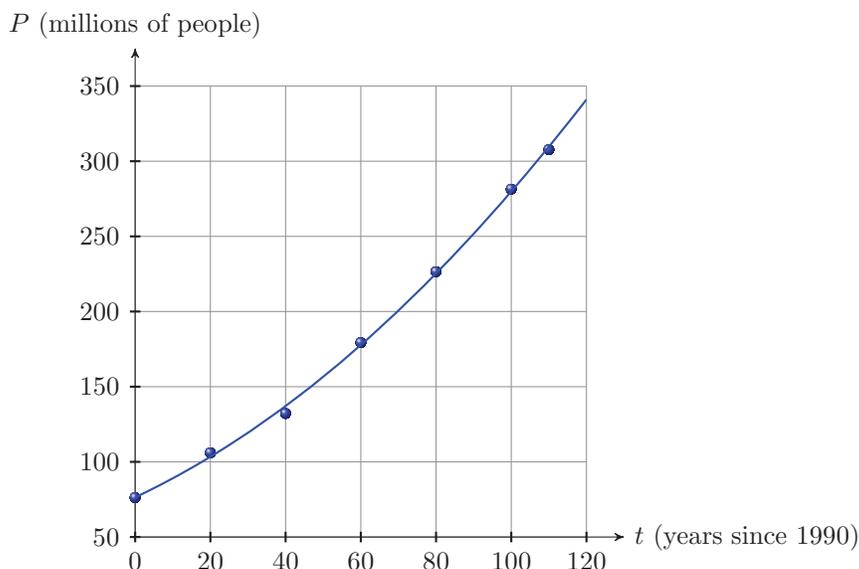
4. The following table records the population (in millions of people) of the United States for the given year.

| Year | 1900 | 1920 | 1940 | 1960 | 1980 | 2000 | 2010 |
|-----------------------|------|-------|-------|-------|-------|-------|-------|
| Population (millions) | 76.2 | 106.0 | 132.2 | 179.3 | 226.5 | 281.4 | 307.7 |

The data is plotted then fitted with the following second degree polynomial, where t is the number of years that have passed since 1990 and $P(t)$ is the population (in millions) t years after 1990.

$$P(t) = 0.008597t^2 + 1,1738t + 76.41$$

Use the graph and then the polynomial to estimate the the population of the United States in the year 1970.



5. If a projectile is launched with an initial velocity of 457 meters per second (457 m/s) from a rooftop 75 meters (75 m) above ground level, at what time will the projectile first reach a height of 6592 meters (6592 m)? Round your answer to the nearest second. *Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8 m/s^2).*
6. If a projectile is launched with an initial velocity of 236 meters per second (236 m/s) from a rooftop 15 meters (15 m) above ground level, at what time will the projectile first reach a height of 1838 meters (1838 m)? Round your answer to the nearest second. *Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8 m/s^2).*
7. If a projectile is launched with an initial velocity of 229 meters per second (229 m/s) from a rooftop 58 meters (58 m) above ground level, at what time will the projectile first reach a height of 1374 meters (1374 m)? Round your answer to

the nearest second. *Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8 m/s^2).*

8. If a projectile is launched with an initial velocity of 234 meters per second (234 m/s) from a rooftop 16 meters (16 m)

above ground level, at what time will the projectile first reach a height of 1882 meters (1882 m)? Round your answer to the nearest second. *Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8 m/s^2).*

In Exercises 9-12, first use an algebraic technique to find the zero of the given function, then use the **2:zero** utility on your graphing calculator to locate the zero of the function. Use the *Calculator Submission Guidelines* when reporting the zero found using your graphing calculator.

9. $f(x) = 3.25x - 4.875$

11. $f(x) = 3.9 - 1.5x$

10. $f(x) = 3.125 - 2.5x$

12. $f(x) = 0.75x + 2.4$

-
13. If a projectile is launched with an initial velocity of 203 meters per second (203 m/s) from a rooftop 52 meters (52 m) above ground level, at what time will the projectile return to ground level? Round your answer to the nearest tenth of a second. *Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8 m/s^2).*

15. If a projectile is launched with an initial velocity of 276 meters per second (276 m/s) from a rooftop 52 meters (52 m) above ground level, at what time will the projectile return to ground level? Round your answer to the nearest tenth of a second. *Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8 m/s^2).*

14. If a projectile is launched with an initial velocity of 484 meters per second (484 m/s) from a rooftop 17 meters (17 m) above ground level, at what time will the projectile return to ground level? Round your answer to the nearest tenth of a second. *Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8 m/s^2).*

16. If a projectile is launched with an initial velocity of 204 meters per second (204 m/s) from a rooftop 92 meters (92 m) above ground level, at what time will the projectile return to ground level? Round your answer to the nearest tenth of a second. *Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8 m/s^2).*

   **Answers**   

1. Approximately \$7,454,000

9. Zero: 1.5

3. Approximately 63 mg/L

11. Zero: 2.6

5. 17.6 seconds

13. 41.7 seconds

7. 6.7 seconds

15. 56.5 seconds