Math 30 **Midterm Review** Name: ____________________
Spring 2009 **D. Arnold**

Instructions. (14 points) Use proper mathematical notation, sound writing, and good organization to provide solutions to each of the following exercises in the space provided. Calculators are not allowed on this examination.

(1^{pt}) **1.** Find all real solutions, if any, of the equation

$$\frac{9}{x-5}+\frac{5}{x-6}=-1$$

Solution: The least common denominator (LCD) is $(x-5)(x-6)$, so first clear fractions by multiplying both sides of the equation by the LCD to obtain the equation

$$9(x-6)+5(x-5)=-1(x-5)(x-6)$$

Then simplify this quadratic equation to standard form:

$$-x^2-3x+49=0$$

Finally, solve this quadratic equation by using the quadratic formula to obtain the solutions $\frac{-3+\sqrt{205}}{2}$ and $\frac{-3-\sqrt{205}}{2}$. Since each of these values does not cause division by zero in the original equation, they are both valid solutions.

(1^{pt}) **2.** Find all real solutions, if any, of the equation

$$\sqrt{x+6}+x=-6$$

Solution:

$$\begin{aligned} &\sqrt{x+6}+x=-6\\ \Longrightarrow\ &\sqrt{x+6}=-x-6\\ \Longrightarrow\ &x+6=(-x-6)^2\\ \Longrightarrow\ &x+6=x^2+12x+36\\ \Longrightarrow\ &x^2+11x+30=0 \end{aligned}$$

Solving this quadratic equation yields $x=-5,-6$.
However, -5 does not solve the original equation.

(1^{pt}) **3.** Find all real solutions, if any, of the equation

$$6x^{2/3}-5x^{1/3}-4=0$$

Solution: Let $u=x^{1/3}$. Then $u^2=(x^{1/3})^2=x^{2/3}$. Substitute these in

$$6x^{2/3}-5x^{1/3}-4=0$$

to obtain

$$6u^2 - 5u - 4 = 0.$$

Factor.

$$(3u - 4)(2u + 1) = 0$$

Thus,

$$u = \frac{4}{3} \quad \text{or} \quad u = -\frac{1}{2}.$$

Substitute back $u = x^{1/3}$.

$$x^{1/3} = \frac{4}{3} \quad \text{or} \quad x^{1/3} = -\frac{1}{2}$$

Raise each equation to the third power.

$$\left(x^{1/3}\right)^3 = \left(\frac{4}{3}\right)^3 \quad \text{or} \quad \left(x^{1/3}\right)^3 = \left(-\frac{1}{2}\right)^3$$

Thus,

$$x = \frac{64}{27} \quad \text{or} \quad x = -\frac{1}{8}.$$

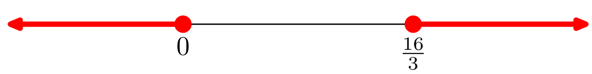
Both answers check.

(1pt) **4.** Solve the inequality: $|3x - 8| \geq 8$. Sketch your answer on a number line, the describe your answer in interval notation.

Solution:

$$\begin{aligned} & |3x - 8| \geq 8 \\ \Longrightarrow \quad & 3x - 8 \geq 8 \quad \text{or} \quad 3x - 8 \leq -8 \\ \Longrightarrow \quad & x \geq \tfrac{16}{3} \quad \text{or} \quad x \leq 0 \end{aligned}$$

In interval notation, the solution is $(-\infty, 0] \cup [\frac{16}{3}, \infty)$.



(1pt) **5.** What is the domain of the function $f(x) = \sqrt{92x - 27}$? Express your answer in interval notation.

Solution: The square root of a negative number is not defined as a real number. Thus, $92x - 27$ must be greater than or equal to zero. $92x - 27 \geq 0$ implies that $x \geq \frac{27}{92}$, so the domain is the interval $\left[\frac{27}{92}, \infty\right)$.

(1^{pt}) **6.** Given the function $f(x) = x^2 - 6x + 1$, find the average rate of change of f from 3 to x, and simplify your answer.

Solution: The average rate of change from 3 to x is the difference quotient

$$\frac{f(x) - f(3)}{x - 3} = \frac{(x^2 - 6x + 1) - ((3)^2 - 6(3) + 1)}{x - 3}$$
$$= \frac{(x^2 - 6x + 1) - (-8)}{x - 3}$$
$$= \frac{x^2 - 6x + 9}{x - 3}.$$

Factor and cancel.

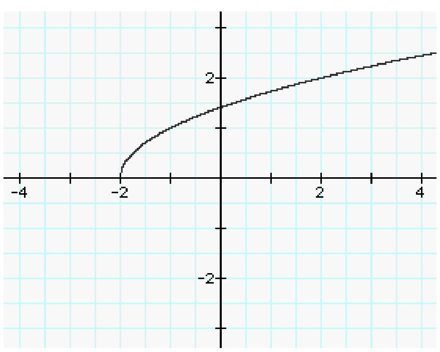
$$= \frac{(x - 3)(x - 3)}{x - 3}$$
$$= x - 3$$

Hence,

$$\frac{f(x) - f(3)}{x - 3} = x - 3,$$

provided $x \neq 3$.

(1^{pt}) **7.** Given the graph of $y = f(x)$ below, sketch the graph of $y = -2f(x) - 1$.



Solution: No solution provided

(1^{pt}) **8.** Find the range of the function $f(x) = x^2 + 8x - 7$. Express your answer in interval notation.

Solution: The graph opens upward since $a = 1 > 0$, and the vertex is at (h, k), where $h = -\frac{b}{2a} = -4$ and $k = f(h) = f(-4) = -23$. Thus, the range is $[k, \infty) = [-23, \infty)$.

(1pt) **9.** Given

$$f(x) = (x-4)(x-8)^4(x-13),$$

Sketch the graph of $f(x)$. Shade the solution of $f(x) > 0$ on the horizontal axis, then describe your solution using interval notation.

Solution: $f(x)$ is a polynomial of *even* degree with leading coefficient 1. Therefore, the graph is *above* the x-axis on the interval $(-\infty, 4)$. The graph then *crosses* the x-axis at zeros of odd multiplicity ($x = 4$ and $x = 13$ in this case), and just *touches* the x-axis at zeros of even multiplicity ($x = 8$). Therefore, the graph is *above* the x-axis on the set

$$(-\infty, 4) \cup (13, \infty)$$

(1pt) **10.** Given

$$f(x) = \frac{x^2 - 5x - 36}{5x - 45},$$

sketch the graph of f. Label all asymptotes with their equations and any critical points with their coordinates.

Solution: Vertical asymptotes occur where the *simplified* function is not defined.

$$\frac{x^2 - 5x - 36}{5x - 45} = \frac{(x-9)(x+4)}{5(x-9)} = \frac{x+4}{5}$$

so there are no vertical asymptotes.

(1pt) **11.** Given that $x = -3$ is a root of $p(x) = x^3 - 6x^2 - 24x + 9$, list all roots of the polynomial $p(x)$.

Solution: Given that $x = -3$ is a root of $p(x) = x^3 - 6x^2 - 24x + 9$, we use synthetic division to find:

$$\begin{array}{r|rrrr} \boxed{-3} & 1 & -6 & -24 & 9 \\ & & -3 & 27 & -9 \\ \hline & 1 & -9 & 3 & 0 \end{array}$$

Thus, $p(x)$ will factor as follows:

$$p(x) = (x + 3)(x^2 - 9x + 3)$$

Use the quadratic formula to find the roots of the quadratic factor.

$$\begin{aligned} x &= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(3)}}{2(1)} \\ &= \frac{9 \pm \sqrt{69}}{2} \end{aligned}$$

(1pt) **12.** Consider the function

$$f(x) = \frac{x^2 - 9}{x + 1}.$$

Sketch the graph of f. Label all asymptotes with their equations and any critical points with their coordinates.

Solution: Solution here.

(1pt) **13.** Calculate $\dfrac{6 + 5i}{-4 - 4i}$. Express your answer in the form $a + bi$.

Solution: To calculate the quotient of two complex numbers, multiply the numerator and denominator by the complex conjugate of the denominator.

$$\begin{aligned} \frac{6 + 5i}{-4 - 4i} &= \frac{6 + 5i}{-4 - 4i} \cdot \frac{-4 + 4i}{-4 + 4i} \\ &= \frac{-24 + 24i - 20i + 20i^2}{16 - 16i + 16i - 16i^2} \\ &= \frac{(-24 - 20) + (24 - 20)i}{16 + 16} \\ &= \frac{-44 + 4i}{32} \\ &= -\frac{11}{8} + \frac{1}{8}i \end{aligned}$$

(1pt) **14.** Given that $x = -3$ is a root of $p(x) = x^3 + 5x^2 + 32x + 78$, list all roots (real or complex) of the polynomial $p(x)$.

Solution: Given that $x = -3$ is a root of $p(x) = x^3 + 5x^2 + 32x + 78$, we use synthetic division to find:

$$\begin{array}{r|rrrr} \boxed{-3} & 1 & 5 & 32 & 78 \\ & & -3 & -6 & -78 \\ \hline & 1 & 2 & 26 & 0 \end{array}$$

Thus, $p(x)$ will factor as follows:

$$p(x) = (x+3)(x^2 + 2x + 26)$$

Use the quadratic formula to find the roots of the quadratic factor.

$$\begin{aligned} x &= \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(26)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{-100}}{2} \\ &= \frac{-2 \pm 10i}{2} \\ &= -1 \pm 5i \end{aligned}$$