Math 30 **Quiz #3B** Name:____________________

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Instructions. (points) Show all your work in the space provided. Each problem requires an algebraic solution.

(5pts) **1.** Find all real solutions, if any, of the equation

$$2x^{2/5} - 3x^{1/5} - 2 = 0$$

Solution: Let $u = x^{1/5}$. Then $u^2 = (x^{1/5})^2 = x^{2/5}$. Substitute these in

$$2x^{2/5} - 3x^{1/5} - 2 = 0$$

to obtain

$$2u^2 - 3u - 2 = 0.$$

Factor.

$$(2u + 1)(u - 2) = 0$$

Thus,

$$u = -\frac{1}{2} \quad \text{or} \quad u = 2.$$

Substitute back $u = x^{1/5}$.

$$x^{1/5} = -\frac{1}{2} \quad \text{or} \quad x^{1/5} = 2$$

Raise each equation to the fifth power.

$$\left(x^{1/5}\right)^5 = \left(-\frac{1}{2}\right)^5 \quad \text{or} \quad \left(x^{1/5}\right)^5 = 2^5$$

Thus,

$$x = -\frac{1}{32} \quad \text{or} \quad x = 32.$$

Both answers check.

(5pts) **2.** Solve the inequality

$$\frac{x-4}{7} - \frac{3-x}{5} \geq 1$$

Sketch your answer on a number line. Use interval and set-builder notation to describe your solution.

Solution: Start with the inequality

$$\frac{x-4}{7} - \frac{3-x}{5} \geq 1$$

and multiply both sides by the common denominator 35.

$$35\left(\frac{x-4}{7}-\frac{3-x}{5}\right)\geq[1]35$$
$$5(x-4)-7(3-x)\geq 35$$

Distribute.

$$5x-20-21+7x\geq 35$$

Simplify.

$$12x-41\geq 35$$

Therefore,

$$12x\geq 76,$$

and

$$x\geq\frac{19}{3}.$$

In interval notation, the solution is $[19/3,\infty)$.

(5^{pts}) **3.** Find all real solutions, if any, of the equation

$$|x^2+8x-37|=28$$

Solution: Start with

$$|x^2+8x-37|=28$$

then set

$$x^2+8x-37=-28 \quad\text{or}\quad x^2+8x-37=28.$$

Make each side of each equation equal to zero.

$$x^2+8x-9=0 \quad\text{or}\quad x^2+8x-65=0.$$

Factor.

$$(x+9)(x-1)=0 \quad\text{or}\quad (x+13)(x-5)=0.$$

Hence, the solutions are -9, 1, -13, and 5.

(5^{pts}) **4.** Solve the inequality: $|-6x+8|\leq 4$. Sketch your solution on a number line. Use interval and set-builder notation to describe your solution.

Solution:

$$\begin{aligned}
& |-6x+8|\leq 4 \\
\Longrightarrow\quad & -6x+8\leq 4 \quad\text{and}\quad -6x+8\geq -4 \\
\Longrightarrow\quad & x\geq\tfrac{2}{3} \quad\text{and}\quad x\leq 2 \\
\Longrightarrow\quad & \tfrac{2}{3}\leq x\leq 2
\end{aligned}$$

Thus, the solution interval is $[\frac{2}{3},2]$.