Math 30 **Prequiz #7B** Name: ____________________

Spring 2009 **D. Arnold**

Instructions. (20 points) For multiple choice questions, circle the letter corresponding to your answer. For other questions, show all your work in the space provided (if any). All answers must be exact unless otherwise indicated.

(5^{pts}) **1.** Suppose that $f(x) = 8x^3 - 4$. Use an algebraic technique to find the formula for the inverse function $f^{-1}(x)$.

Solution: Start with the equation $y = 8x^3 - 4$ and interchange x and y:

$$x = 8y^3 - 4$$

Then solve for y:

$$\begin{aligned} 8y^3 &= x + 4 \\ y^3 &= \frac{x+4}{8} \\ y &= \sqrt[3]{\frac{x+4}{8}} \end{aligned}$$

Hence,

$$f^{-1}(x) = \sqrt[3]{\frac{x+4}{8}}.$$

(5^{pts}) **2.** If $f(x) = \sqrt{x+8}$ and $g(x) = x^2 + 4$, find the formula for $(g \circ f)(x)$ and simplify your answer as much as possible.

Solution: By definition,

$$(g \circ f)(x) = g(f(x))$$

Because $f(x) = \sqrt{x+8}$, this becomes

$$= g(\sqrt{x+8}).$$

Substitute $\sqrt{x+8}$ into the function $g(x) = x^2 + 4$ and simplify.

$$\begin{aligned} &= (\sqrt{x+8})^2 + 4 \\ &= (x+8) + 4 \\ &= x + 12 \end{aligned}$$

Thus,

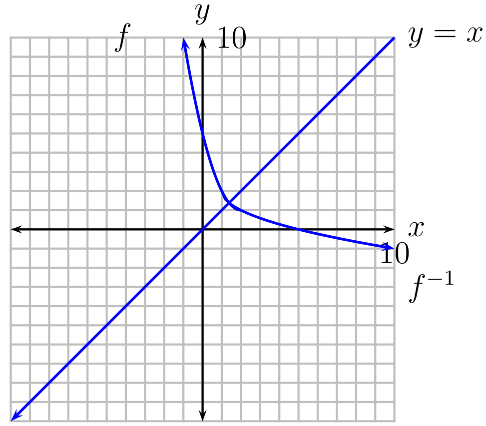
$$(g \circ f)(x) = x + 12.$$

(5[pts]) **3.** Given $f(x) = x^2 - 4x + 5$, $x \leq 2$, sketch the graph of f, the line $y = x$, and the reflection of the graph of f across the line $y = x$. Use these results and algebraic technique to help find the formula for $f^{-1}(x)$.

Solution: First, we need to get the equation in vertex form.

$$f(x) = x^2 - 4x + 4 - 4 + 5$$
$$f(x) = (x-2)^2 + 1$$

Hence, the parabola opens upward, shifted 2 units right, and 1 unit up, as seen in the figure that follows, along with the line $y = x$, and the reflection across the line $y = x$.



Start with the equation $y = x^2 - 4x + 5, \quad x \leq 2$ and interchange x and y: $x = y^2 - 4y + 5$, $y \leq 2$. Then solve for y:

$$\begin{aligned} & x = y^2 - 4y + 5, && y \leq 2 \\ \implies & y^2 - 4y + 5 - x = 0, && y \leq 2 \\ \implies & y = \frac{4 \pm \sqrt{4x-4}}{2}, && y \leq 2 \\ \implies & y = 2 \pm \sqrt{x-1}, && y \leq 2 \end{aligned}$$

Since $y \leq 2$, it follows that $y = 2 - \sqrt{x-1}$. Thus, $f^{-1}(x) = 2 - \sqrt{x-1}$, whose graph does look like the reflection in our graph above.

(5[pts]) **4.** Find all roots (real or complex) of $p(x) = -15x^3 + 28x^2 + 5x - 2$.

Solution: Because the polynomial $p(x) = -15x^3 + 28x^2 + 5x - 2$ has integer coefficients, we can apply the *Rational Root Theorem*, which says that if p/q is a root of $p(x)$, then p divides the constant term -2. Therefore, possibilities for p include

$$\pm 1,\ \pm 2.$$

The *Rational Root Theorem* states that q must divide the leading coefficient -15. Hence, the only possibilities for q are

$$\pm 1,\ \pm 3,\ \pm 5,\ \pm 15.$$

Hence, possible rational roots p/q are

$$\pm 1,\ \pm 1/3,\ \pm 1/5,\ \pm 1/15,\ \pm 2,\ \pm 2/3,\ \pm 2/5,\ \pm 2/15.$$

Using synthetic division, we discover:

$$\begin{array}{c|cccc} & -15 & 28 & 5 & -2 \\ \frac{1}{5} & & -3 & 5 & 2 \\ \hline & -15 & 25 & 10 & 0 \end{array}$$

Therefore, $x = 1/5$ is a root and $x - 1/5$ is a factor of $p(x)$.

$$\begin{aligned} p(x) &= \left(x - \frac{1}{5}\right)(-15x^2 + 25x + 10) \\ &= -5\left(x - \frac{1}{5}\right)(3x^2 - 5x - 2) \end{aligned}$$

Factor the quadratic factor.

$$p(x) = -5\left(x - \frac{1}{5}\right)(3x + 1)(x - 2)$$

Therefore, the roots of $p(x)$ are $x = 1/5$, $x = -1/3$, and $x = 2$.