INTRO TO LaTeX

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September 11, 2009

Abstract

The purpose of this article is to demonstrate my knowledge and use of LaTeX by solving a system of equations.

Introduction

Producing a quality scientific paper on a professional level can be difficult. With the help of software like LaTeX writing such a paper can be done with ease by both the expert and the beginner with LaTeX. In this article I will demonstrate my knowledge from past experiences with using LaTeX and introduce the use of the bmatrix environment [2] while solving a system of equations with three equations and three unknowns.

Solving A System Of Equations

The system

The system has 3 equations and 3 unknowns. The equations are

$$\begin{aligned} x + 2y + 3z &= 6 \\ 2x - y + 4z &= 8 \\ -x + 8y + 2z &= 12. \end{aligned} \tag{1}$$

The augmented matrix

The first step in solving is to place system (1) in augmented matrix form. The augmented matrix is

$$M = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & -1 & 4 & 8 \\ -1 & 8 & 2 & 12 \end{bmatrix}. \tag{2}$$

Row echelon form using Gaussian elimination

The next step is to place matrix M into row echelon form using the Gaussian elimination process. Start by making the first non-zero number from the first row of (2) a pivot number

$$M = \begin{bmatrix} \boxed{1} & 2 & 3 & 6 \\ 2 & -1 & 4 & 8 \\ -1 & 8 & 2 & 12 \end{bmatrix}.$$

The Gaussian elimination allows us to add or subtract multiples of a row to another row in order to create a matrix with zeros beneath the pivot number. Take 2 times row 1 from row 2 and row 1 to row 3 to get

$$M = \begin{bmatrix} \boxed{1} & 2 & 3 & 6 \\ 0 & -5 & -2 & -4 \\ 0 & 10 & 5 & 18 \end{bmatrix}. \tag{3}$$

The next step is to create a new pivot number from row 2 in matrix M by choosing the first non-zero number in row 2.

$$M = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & \boxed{\text{-5}} & -2 & -4 \\ 0 & 10 & 5 & 18 \end{bmatrix}$$

To create a new matrix M with zeros beneath the pivot in row 2 take 2 times row 2 to row 3 to get

$$M = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & \boxed{\text{-5}} & -2 & -4 \\ 0 & 0 & 1 & -10 \end{bmatrix}. \tag{4}$$

The result is a new matrix R in row echelon form.

$$R = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -5 & -2 & -4 \\ 0 & 0 & 1 & 10 \end{bmatrix} \tag{5}$$

Back substitution

In back substitution we take matrix R and form a new system of equations. This system still has 3 equations and 3 unknowns which are

$$x + 2y + 3z = 6 \tag{6}$$
$$-5y - 2z = -4 \tag{7}$$
$$z = 10. \tag{8}$$

The first step is to solve equation (8) for z

$$z = 10. \tag{9}$$

The next step is to substitute $z = 10$ in equation (7) and solve for y. Thus,

$$\begin{aligned} -5y - 2(10) &= -4 \\ -5y - 20 &= -4 \\ -5y &= 16 \\ y &= \frac{-16}{5}. \end{aligned}$$

We now have the second unknown

$$y = \frac{-16}{5}. \tag{10}$$

Finally we substitute $z = 10$ and $y = -16/5$ in equation (8) and solve for x. Thus,

$$\begin{aligned} x + 2\left(\frac{-16}{5}\right) + 3(10) &= 6 \\ x + \frac{-32}{5} + \frac{150}{5} &= \frac{30}{5} \\ x + \frac{118}{5} &= \frac{30}{5} \\ x &= \frac{-88}{5}. \end{aligned}$$

We now have the third and final unknown

$$x = \frac{-88}{5}. \tag{11}$$

Check

Check our work using the original system (1). Remember that system (1) is

$$x + 2y + 3z = 6 \tag{12}$$
$$2x - y + 4z = 8 \tag{13}$$
$$-x + 8y + 2z = 12. \tag{14}$$

First substitute solutions $z = 10$, $y = -16/5$, and $x = -88/5$ in to equation (12)

$$
\begin{aligned}
\frac{-88}{5} + 2\left(\frac{-16}{5}\right) + 3\,(10) &= 6\\
\frac{-88}{5} - \frac{32}{5} + 30 &= 6\\
-24 + 30 &= 6\\
6 &= 6.
\end{aligned}
$$

The solutions $z = 10$, $y = -16/5$, and $x = -88/5$ are a solution for equation (12). Next substitute solutions $z = 10$, $y = -16/5$, and $x = -88/5$ in to equation (13)

$$
\begin{aligned}
2\left(\frac{-88}{5}\right) - \left(\frac{-16}{5}\right) + 4\,(10) &= 8\\
\frac{-176}{5} + \frac{16}{5} + 40 &= 8\\
-32 + 40 &= 8\\
8 &= 8.
\end{aligned}
$$

The solutions $z = 10$, $y = -16/5$, and $x = -88/5$ are a solution for equation (13). Finally substitute solutions $z = 10$, $y = -16/5$, and $x = -88/5$ in to equation (14)

$$
\begin{aligned}
-\frac{-88}{5} + 8\left(\frac{-16}{5}\right) + 2\,(10) &= 12\\
\frac{88}{5} - \frac{128}{5} + 20 &= 12\\
\frac{-40}{5} + 20 &= 12\\
-8 + 20 &= 12\\
12 &= 12.
\end{aligned}
$$

The solutions $z = 10$, $y = -16/5$, and $x = -88/5$ are a solution for equation (14).

References

[1] D. Arnold. *S2009 Math55.*

[2] D. Arnold. *2009 Introduction to LaTeX.*

[3] D. Arnold. *2008 Writing Scientific Papers in LaTeX.*