Math 45 Assignment #1
Introduction to LaTeX

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Abstract

Assignment 1 for Math 45 Linear Algebra will be to take a given system of linear equations, set up an augmented matrix for the system, and use Gaussian elimination to place the augmented matrix in row echelon form. Back substitution will then be used to solve the original system of linear equations.

Introduction

Given the system of linear equations

$$\begin{aligned} x + 2y + 3z &= 6 \\ 2x - y + 4z &= 8 \\ -x + 8y + 2z &= 12 \end{aligned} \tag{1}$$

solve for x, y, z. This can be done in linear algebra through the equation $A\vec{x} = \vec{b}$, where matrix A is the coefficient matrix as described in the next section, the unknown $\vec{x}$ is a column vector containing each variable (x, y, z for this example), and $\vec{b}$ is also a column vector which contains the right side of the system of equations.

Solving Linear Equations

The first step to solving a system of linear equations is to create a coefficient matrix. A coefficient matrix is exactly as it sounds; a matrix filled with coefficients. Each element in the coefficient matrix corresponds to the coefficient of the linear equation it is being produced from. Here is the coefficient matrix for the system (1).

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ -1 & 8 & 2 \end{bmatrix}$$

Combining this with our equation for solving a system of linear equations, $A\vec{x} = \vec{b}$ becomes:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ -1 & 8 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$$

By adding $\vec{b}$ into the final column of A we have created an augmented matrix. An augmented matrix is the coefficient matrix with the $\vec{b}$ added as the final column. The augmented matrix for (1) is matrix A with $\vec{b}$ added into the fourth column.

$$A = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & -1 & 4 & 8 \\ -1 & 8 & 2 & 12 \end{bmatrix} \tag{2}$$

From here we can now use Gaussian elimination to solve the system.

Gaussian Elimination

Once an augmented matrix is created from the system, elimination can be performed with the goal of creating an upper triangular system with non zero coefficients running down the diagonal of the triangle. Once completed, this new system will be in row echelon form. This is achieved through elimination.

Elimination is the process of subtracting multiples of rows within the matrix from other rows to create columns of zeroes below each pivot. A pivot is the first nonzero in the row that does the elimination and the multiplier l is the entry being eliminated divided by the pivot. To find the multiplier, divide the coefficient of the row element to be eliminated by the pivot of the row which will be doing the elimination. The pivots are on the diagonal of the upper triangular system after elimination.

Elimination will now be conducted to put (2) into row echelon form. The first pivot is A_{11}, with multiplier $l = 2/1 = 2$. $R_2 - 2R_1$ will be read as "Row two subtract two times row one".

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & -1 & 4 & 8 \\ -1 & 8 & 2 & 12 \end{bmatrix}$$

Row two subtract two times to row one.

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -5 & -2 & -4 \\ -1 & 8 & 2 & 12 \end{bmatrix}$$

Row three subtract negative one times row one.

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -5 & -2 & -4 \\ 0 & 10 & 5 & 18 \end{bmatrix}$$

Row three subtract negative two times row two.

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -5 & -2 & -4 \\ 0 & 0 & 1 & 10 \end{bmatrix} \tag{3}$$

The system is now in row echelon form.

Solving the Row Echelon System

Rewriting the matrix as a system of linear equations represented by (3) is

$$\begin{aligned} x + 2y + 3z &= 6 && (4) \\ -5y - 2z &= -4 && (5) \\ z &= 10 && (6) \end{aligned}$$

From here, we can use back substitution to solve the system of equations. From equation (6) it is known that $z = 10$. Substituting $z = 10$ into (5).

$$\begin{aligned} -5y - 2(10) &= -4 \\ -5y &= 16 \\ y &= -\frac{16}{5} \end{aligned}$$

Substituting $y = -16/5$ into (4)

$$\begin{aligned} x + 2\left(-\frac{16}{5}\right) + 3(10) &= 6 \\ x - \frac{32}{5} + 30 &= 6 \\ x - \frac{32}{5} &= -24 \\ x &= -\frac{88}{5} \end{aligned}$$

Next we will substitute x, y, and z back into the original system (1).

$$\begin{aligned} \left(-\frac{88}{5}\right) + 2\left(-\frac{16}{5}\right) + 3(10) &= 6 \\ 2\left(-\frac{88}{5}\right) - \left(-\frac{16}{5}\right) + 4(10) &= 8 \\ -\left(-\frac{88}{5}\right) + 8\left(-\frac{16}{5}\right) + 2(10) &= 12 \end{aligned}$$

Complete each multiplication.

$$
\begin{aligned}
-\frac{88}{5} - \frac{32}{5} + 30 = 6 \\
-\frac{176}{5} + \frac{16}{5} + 40 = 8 \\
\frac{88}{5} - \frac{122}{5} + 20 = 12
\end{aligned}
$$

Adding terms together to complete each equation.

$$
\begin{aligned}
-\frac{120}{5} + 30 = 6 \\
-\frac{160}{5} + 40 = 8 \\
-\frac{32}{5} + 20 = 12
\end{aligned}
$$

$$
\begin{aligned}
6 = 6 \\
8 = 8 \\
12 = 12
\end{aligned}
$$

By back substitution we end up with three true statements. The process of elimination has solved our system of linear equations.

References

[1] D. Arnold. *Writing Scientific Papers in LaTeX.*