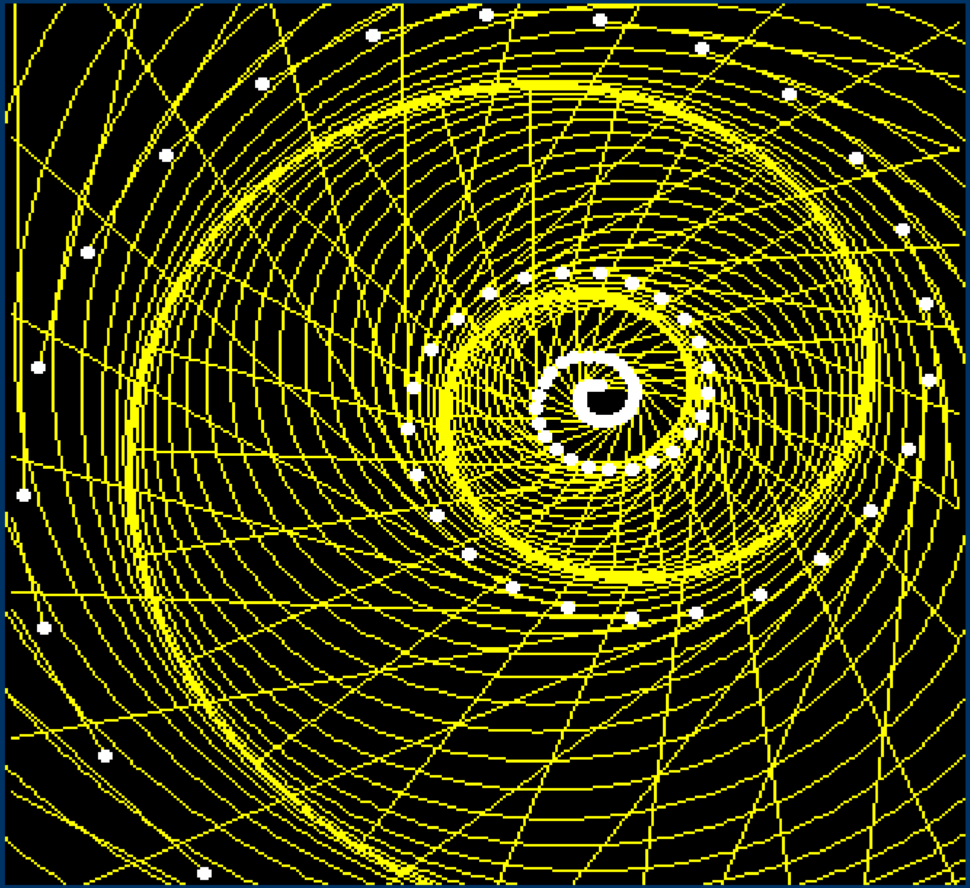
Special Plane Curves

Equiangular Spirals



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History

The investigation of spirals began with the ancient Greeks

The equiangular spiral was first considered in 1638 by Rene Descartes, who started from the property s = ar.

The spiral properties of self-reproduction were discovered by Jacob Bernoulli (1654-1705)

DEFINITION

The logarithmic spiral is a spiral whose polar equation is given by

$$r=ae^{b\theta},$$

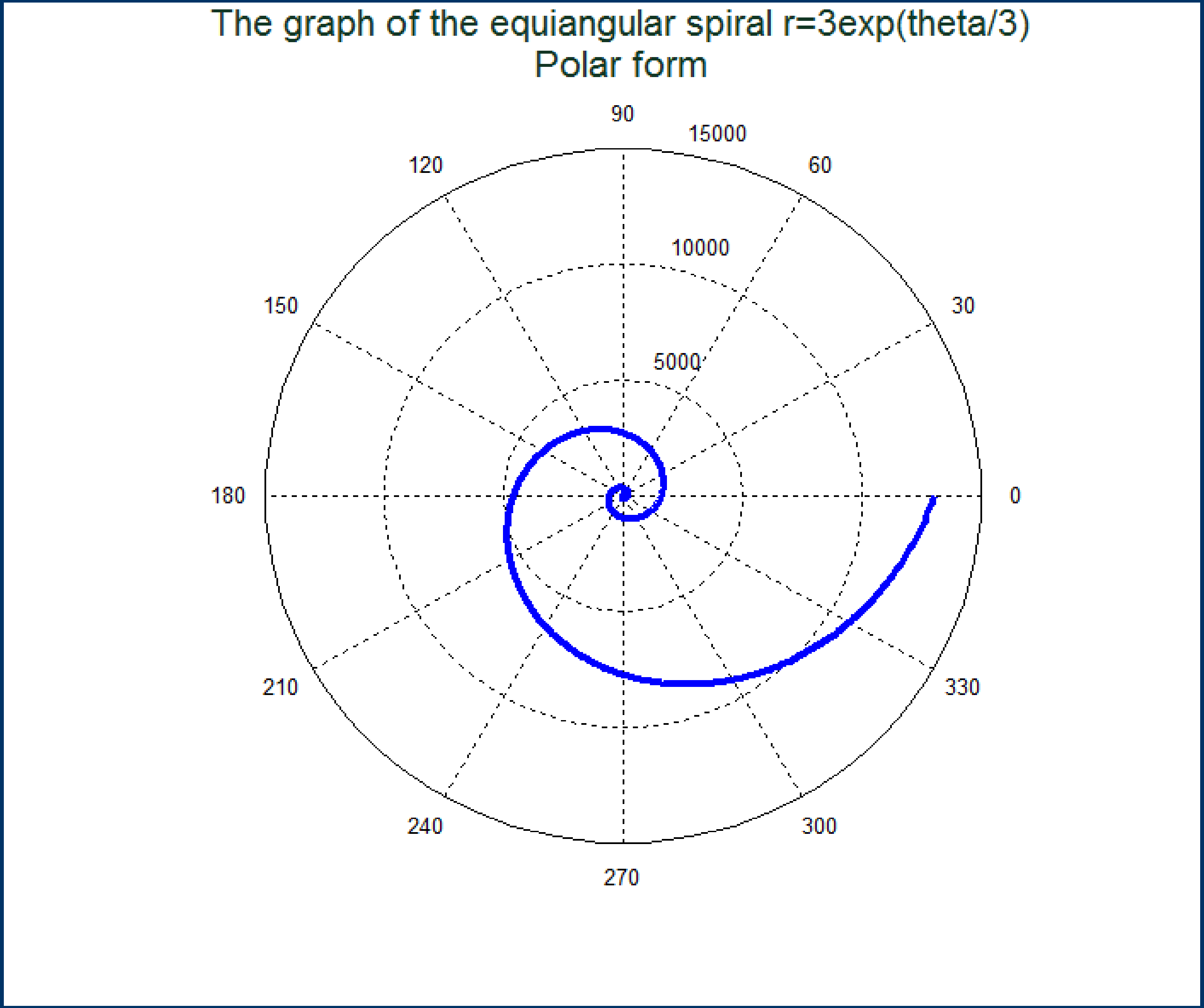
where r is the distance from the origin, theta is the angle from the x-axis, and a and b are arbitrary constants. The logarithmic spiral is also known as the growth spiral, equiangular spiral, and spira mirabilis. It can be expressed parametrically as

$$x = r\cos\theta = a\cos\theta e^{b\theta}$$

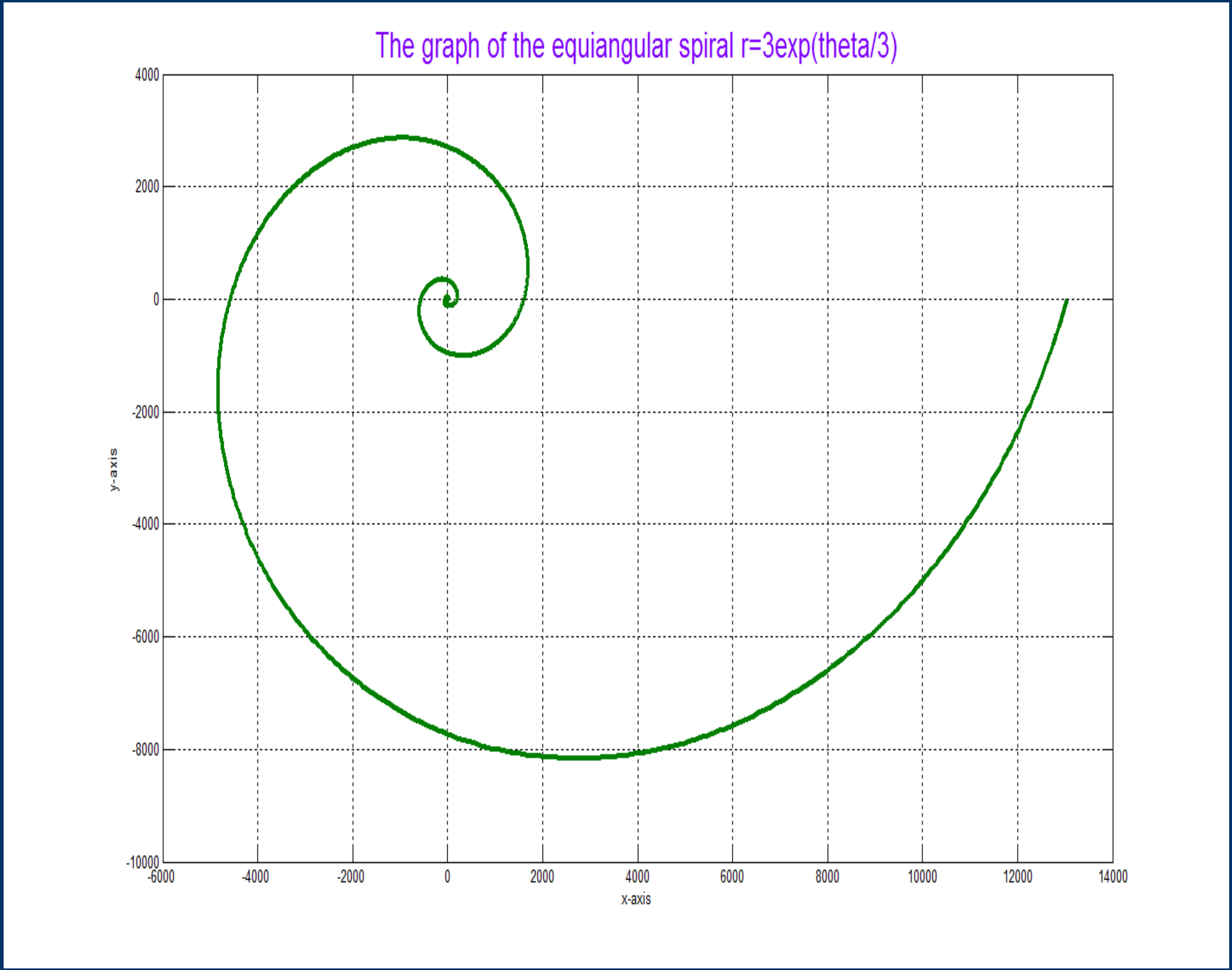
$$y = r\sin\theta = a\sin\theta e^{b\theta}$$

Equiangular spiral describes a family of spirals of one parameter. It is defined as a curve that cuts all radial line at a constant angle.

It also called logarithmic spiral, Bernoulli spiral, and logistique.



The graph of the equiangular spiral r=3exp(theta/3)
Polar form
90
15000
120
60
10000
150
30
5000
180
0
210
330
240
300
270



The graph of the equiangular spiral r=3exp(theta/3)
4000
2000
0
-2000
-4000
-6000
-8000
-10000
y-axis
-6000
-4000
-2000
0
2000
4000
6000
8000
10000
12000
14000
x-axis

Example

Consider the polar function $r=ae^{b\theta}$

Let a = 2.2 and b = 1.7 radians

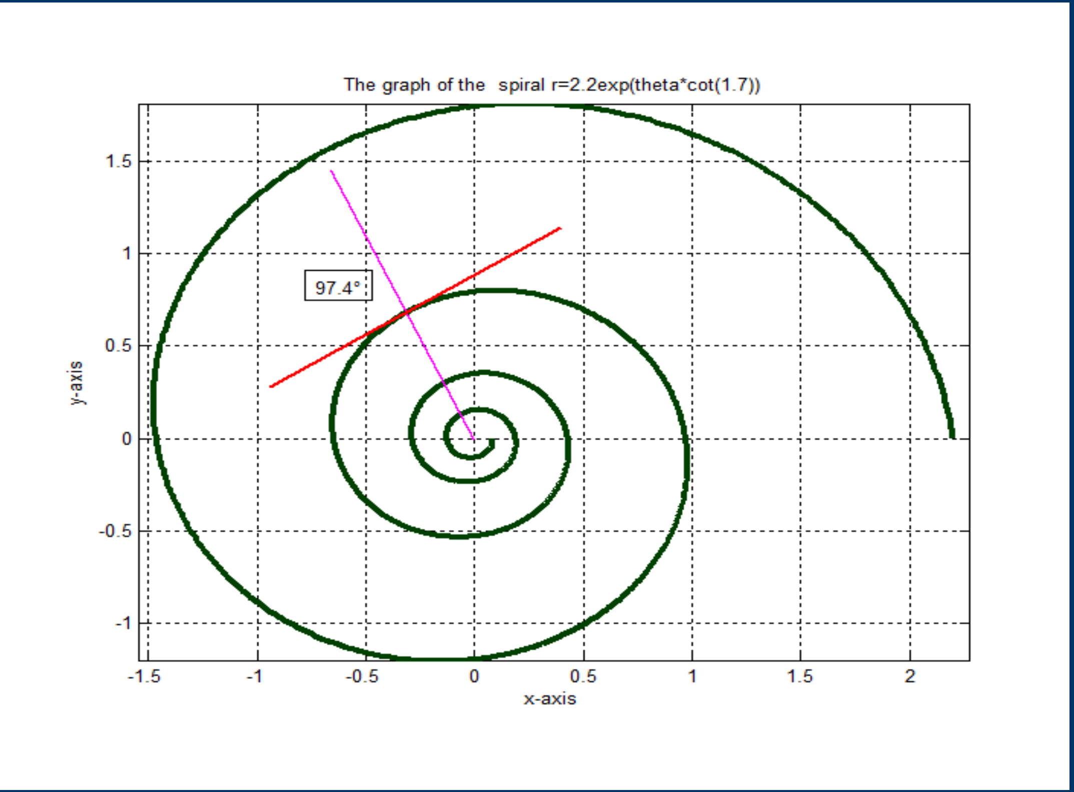
Then

$r = 2.2\, e^{\theta \cot 1.7}$

In this example, b = 1.7 radians, or

$$1.7\frac{180}{\pi} = 97.4^\circ$$

Using Matlab we have



As we can see the angle between the radial line and the tangent line is 97.4°

Now I will use Geogebra to demonstrate that in equiangular spirals the angle between the radial line and the tangent line to the curve is always constant at any point .

1. Lets start with the parametric equations

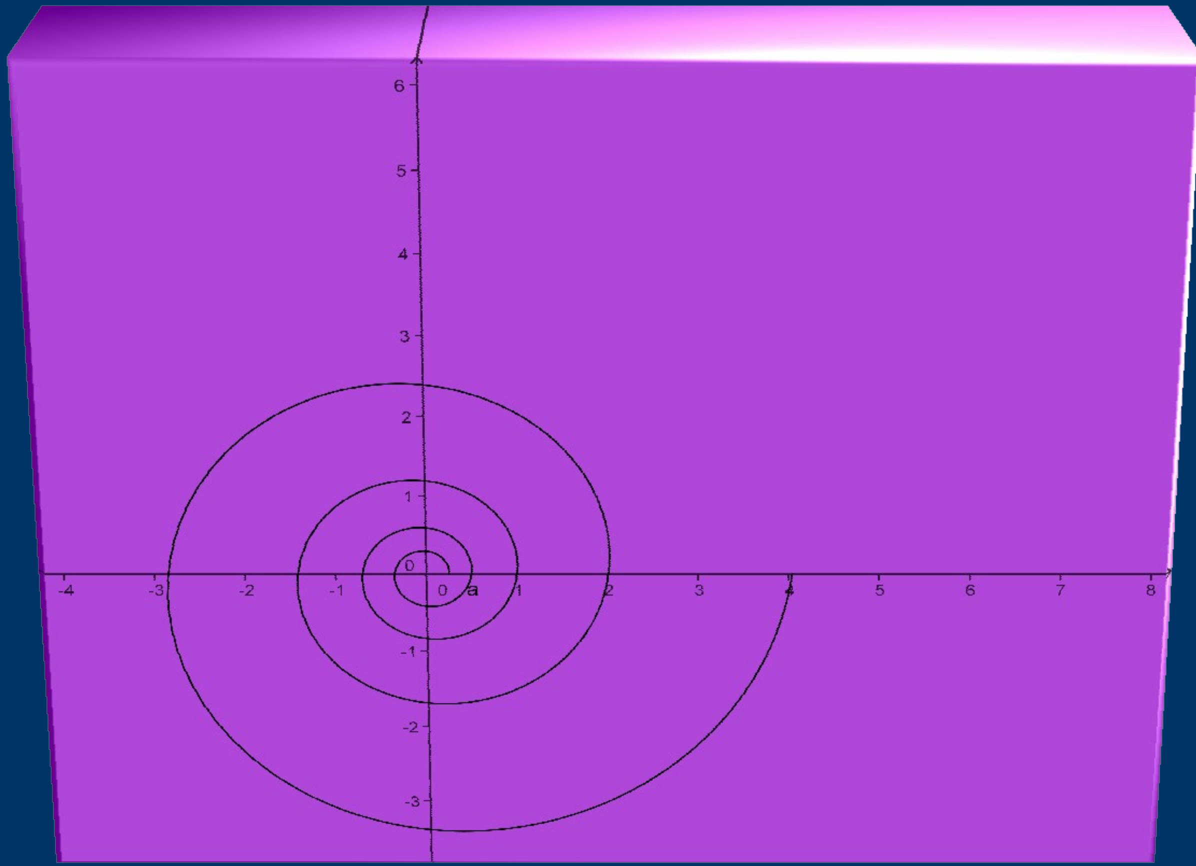
$$x=e^{t/\tan(\alpha)\cos(t)}$$

$$y=e^{t/\tan(\alpha)\sin(t)}$$

where α is an arbitrary angle

and $-4\pi < t < 4\pi$

In this case α = 80°



2. Then lets draw a radial line from the center of the spiral through an arbitrary point on the curve

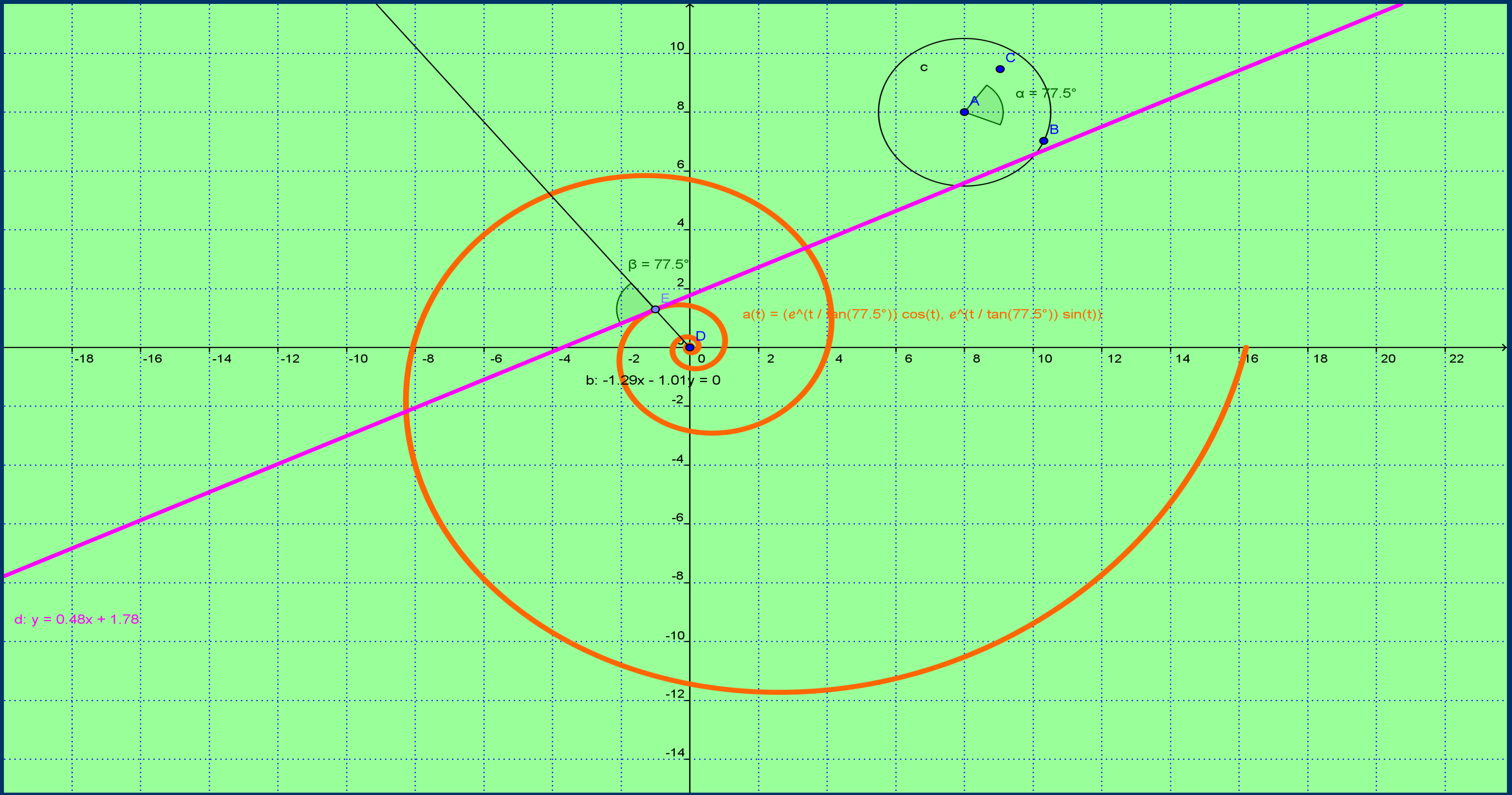
3. then we draw a tangent line at the arbitrary point

4. If the angle formed by the radial line and the tangent at any point is constant, the spiral is equiangular

Then we will obtain the following image

Equiangular Spiral

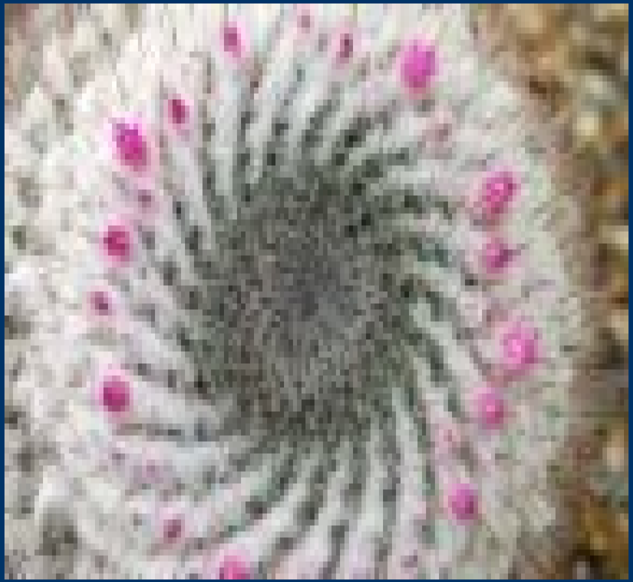
Alfredo Lopez, Created with GeoGebra



Click the hyperlink to open the active worksheet

K:\Math 50 C Final Project\Equiangular Spiral 2.html

This spiral is related to Fibonacci numbers, the golden ratio, and golden rectangles, and is sometimes called the golden spiral.



Spiral in nature

References

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[3] xahlee.org. “Equiangular Spiral”.
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http://www.intmath.com/Plane-analytic-geometry/Equiangular_spiral.php