Math 50C **Quiz #5A** Name: ________________

Fall 2009 **D. Arnold**

Instructions. (20 points) Answer each of the following questions in the space provided.

(5pts) **1.** State the definition for $f_y(x, y)$, then use the definition to find the first partial of $f(x, y) = x^2 + y^2 - 9$ with respect to y. Show all of the steps and work in the space provided below.

Solution:

$$\begin{aligned} f_y(x, y) &= \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h} \\ &= \lim_{h \to 0} \frac{[x^2 + (y+h)^2 - 9] - [x^2 + y^2 - 9]}{h} \\ &= \lim_{h \to 0} \frac{(y+h)^2 - y^2}{h} \\ &= \lim_{h \to 0} \frac{y^2 + 2yh + h^2 - y^2}{h} \\ &= \lim_{h \to 0} \frac{2yh + h^2}{h} \\ &= \lim_{h \to 0} 2y + h \\ &= 2y \end{aligned}$$

(5pts ea.) **2.** Consider the function $f(x, y) = x^3 - 2xy - 3y^2 - 9$.

(a) Find $\dfrac{\partial^2 f}{\partial x^2}$.

Solution:

$$\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \\ &= \frac{\partial}{\partial x}\left(3x^2 - 2y\right) \\ &= 6x \end{aligned}$$

(b) Find $\dfrac{\partial^2 f}{\partial x \partial y}$.

Solution:

$$\begin{aligned} \frac{\partial^2 f}{\partial x \partial y} &= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \\ &= \frac{\partial}{\partial x}\left(-2x - 6y\right) \\ &= -2 \end{aligned}$$

(5pts) **3.** Consider the equation $x^3 + y^3 + z^3 = 3xyz$. Use implicit differentiation to find z_y.
Solution:

$$x^3 + y^3 + z^3 = 3xyz$$

Differentiate with respect to y.

$$3y^2 + 3z^2 z_y = 3x\frac{\partial}{\partial y} yz$$

Use the product rule.

$$3y^2 + 3z^2 z_y = 3x\left[y z_y + z\right]$$

Finish with some algebra.

$$\begin{aligned} 3y^2 + 3z^2 z_y &= 3xyz_y + 3xz \\ (3z^2 - 3xy) z_y &= 3xz - 3y^2 \end{aligned}$$

Divide both sides by 3.

$$\begin{aligned} (z^2 - xy) z_y &= xz - y^2 \\ z_y &= \frac{xz - y^2}{z^2 - xy} \end{aligned}$$