The Para-Bungee

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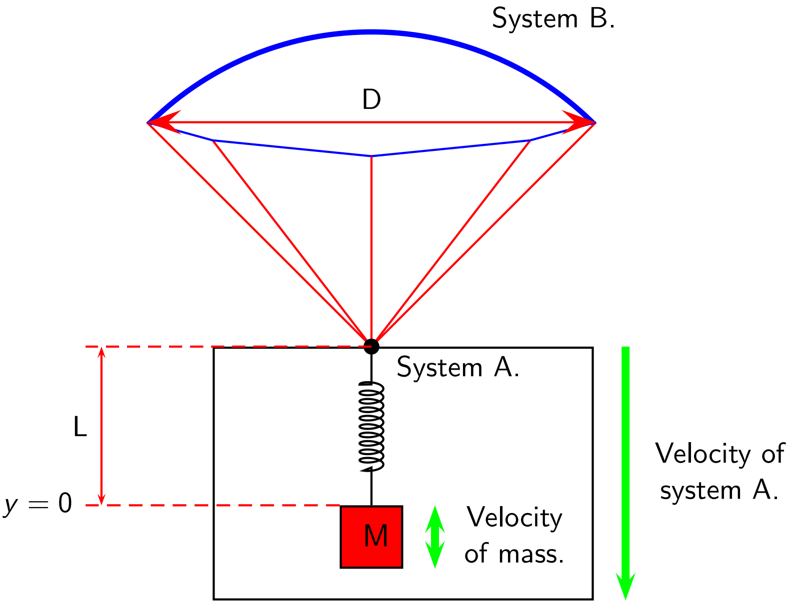
Why would a parabungee be used?

- This idea will allow high decent rates with minimal velocity landings.
- Safely delivering wide range of packages with near zero impact damage.
- Smaller parachutes may be used as a result of this.

Now lets examine the principal components of the Para-Bungee.

- I chose to analyze it in two parts, System A and System B.
- System A moves within System B.
- We will examine System A first then show how it moves with in System B allowing a zero velocity landing.
- Now we will have a look at a diagram of the both systems to gain an understanding of how it operates.


System B.
D
System A.
L
$y = 0$
M
Velocity
of mass.
Velocity of
system A.

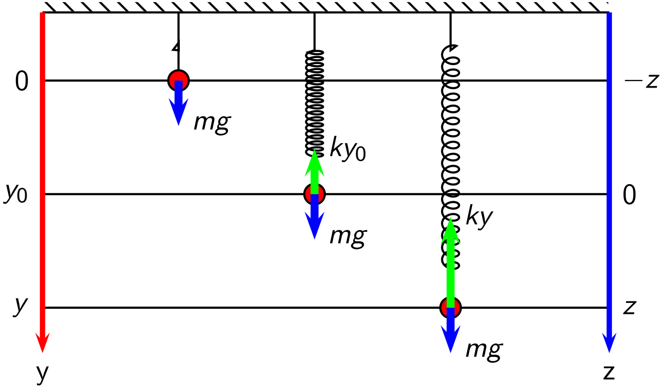
The basic kinematics of the Para-Bungee

- As we can see the cargo is attached to the parachute by a bungee chord or better known as a spring mass system.
- As the parachute and cargo descends after time zero as defined for this problem the cargo is free to move relative to the parachute.
- In order to understand the translational motion we must look at a spring mass system.
- But first we need to define our axis and variables.

Defining axis direction and variables

$$\begin{aligned}
\sum F &= ma \\
ma &= mg - ky \\
my'' &= ky_0 - ky \\
my'' &= k(y_0 - y)
\end{aligned}$$

A typical spring mass system



y in terms of z

$$z = y - y_0$$
$$z' = y'$$
$$z'' = y''$$

Derivation of translational motion from the Spring mass diagram

We are dealing with a oscillator so our spring mass system will be expressed as

$$mz'' + \mu z' + kz = 0$$

Since we are assuming an undamped system we may remove the middle term which is the damping term.

$$mz'' + kz = 0$$

Derivatives of z in terms of $e^{\lambda t}$

We now let $z = e^{\lambda t}$

$$z = e^{\lambda t}$$

$$z' = \lambda e^{\lambda t}$$

$$z'' = \lambda^2 e^{\lambda t}$$

Solving for Lambda

Replacing the z terms in the undamped oscillator we will solve for lambda

$$m\lambda^2 e^{\lambda t} + ke^{\lambda t} = 0$$

$$m\lambda^2 + k = 0$$

$$\lambda^2 = -\frac{k}{m}$$

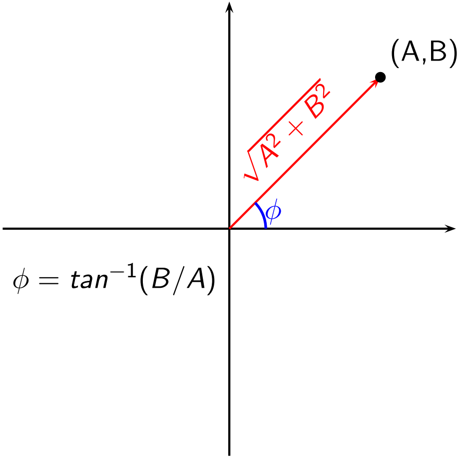
$$\lambda = \pm\sqrt{k/m}i$$

Solving the system

Next we will smplify solution

$$
\begin{aligned}
z =& e^{\sqrt{k/m}(it)} \\
=& \cos\left(\sqrt{k/m} * t\right) + i \sin\left(\sqrt{k/m} * t\right) \\
=& A \cos\left(\sqrt{k/m} * t\right) + B \sin\left(\sqrt{k/m} * t\right) \\
=& \sqrt{A^2 + B^2}\left[\frac{A}{\sqrt{A^2 + B^2}} \cos\left(\sqrt{\frac{k}{m}} t\right) + \frac{B}{\sqrt{A^2 + B^2}} \sin\left(\sqrt{\frac{k}{m}} t\right)\right]
\end{aligned}
$$

The definition of ϕ



The final solution for z

$$
\begin{aligned}
z &= \sqrt{A^2 + B^2}\left[\cos(\phi)\cos\left(\sqrt{k/m} * t\right) + \sin(\phi)\sin\left(\sqrt{k/m} * t\right)\right] \\
&= \sqrt{A^2 + B^2}\left[\cos\left(\sqrt{k/m} * t - \phi\right)\right] \\
z' &= -\sqrt{A^2 + B^2}\sqrt{k/m}\left[\sin\left(\sqrt{k/m} * t - \phi\right)\right]
\end{aligned}
$$

Defining the constants

Going up to the definition of phi we will see that,

$$\phi = \tan^{-1}(B/A).$$

Now we need A and B to get them we need to define $z(0)$ and $z'(0)$. Since we will start with the bungee tight but unstreched,

$$z(0) = -\frac{mg}{k}$$

Because the system starts with the mass stationary with in it's frame of reference,

$$z'(0) = 0.$$

To get A and B we insert these values of z and z' into,

$$\begin{aligned} z &= A\cos\left(\sqrt{k/m}t\right) + B\sin\left(\sqrt{k/m}t\right) \\ -\frac{mg}{k} &= A\cos(0) \\ A &= -\frac{mg}{k} \end{aligned}$$

$$\begin{aligned} z' &= -A\sqrt{k/m}\sin\left(\sqrt{k/m}t\right) + B\sqrt{k/m}\cos\left(\sqrt{k/m}t\right) \\ 0 &= B\sqrt{k/m}\cos(0) \\ B &= 0 \end{aligned}$$

Now we can solve for the velocity of the parachute

$$v = 29 \times \sqrt{\frac{F}{1.4 \times \pi r^2}}$$

The force will be defined by

$$F = k\left(z - \frac{mg}{k}\right) = kz - mg$$

$$v = 29 \times \sqrt{\frac{kz - mg}{1.4 \times \pi r^2}}$$

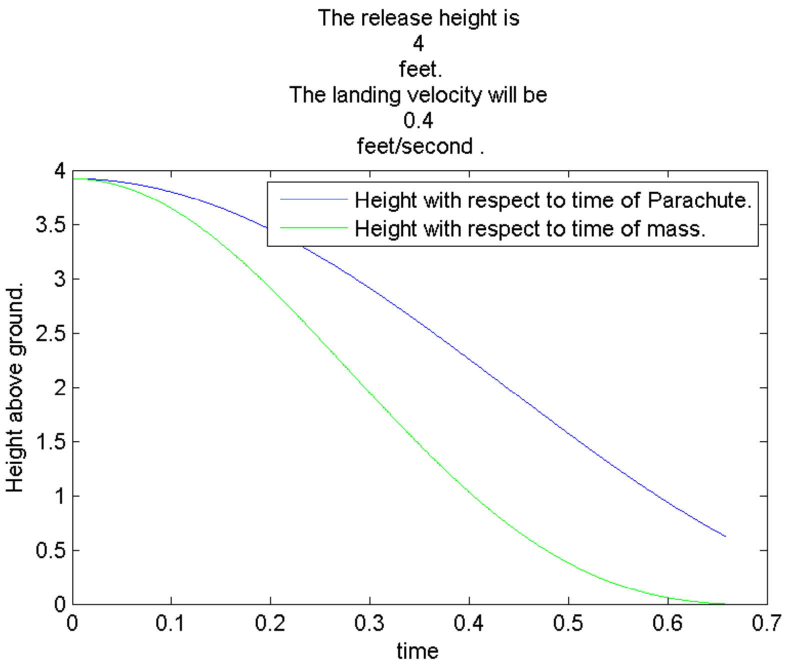
Where z is defined by,

$$z = -\frac{mg}{k} \cos\left(\sqrt{k/m} * t\right)$$

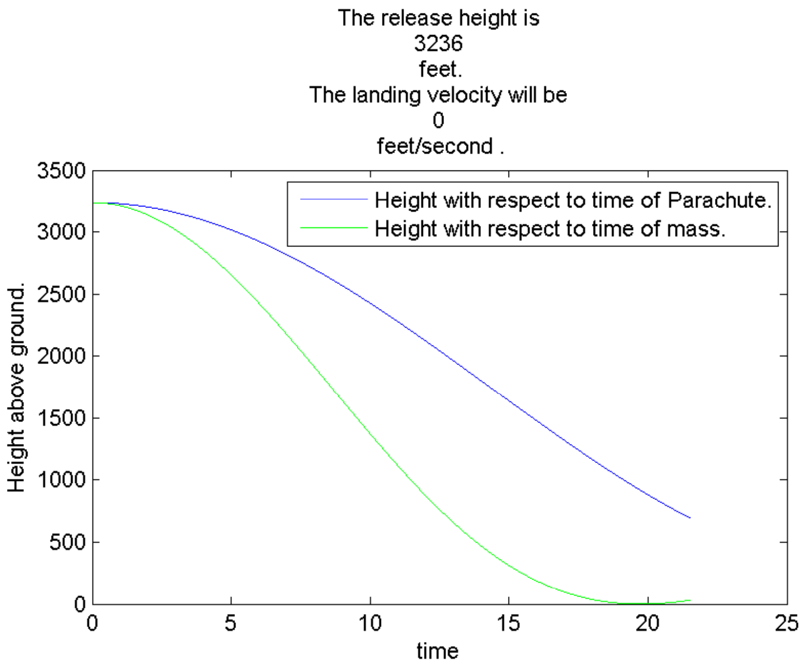
Finally lets display all of the neccacary results

$$
\begin{aligned}
v &= 29 \times \sqrt{\frac{kz - mg}{1.4 \times \pi r^2}} \\
z &= -\frac{mg}{k} \cos\left(\sqrt{k/m} * t\right) \\
z' &= g\sqrt{m/k} \sin\left(\sqrt{k/m} t\right)
\end{aligned} \tag{1}
$$

The weight of the load in lbs = .125 . The stretch in lbs per foot applied to the bungee = .2 . The diameter of the parachute in feet = 2 . The release height is 3.91 feet off of the ground. Landing velocity feet/sec if bungee had not been deployed 4.89 . Landing velocity in feet/sec with bungee deployment 0.42 .


The release height is
4
feet.
The landing velocity will be
0.4
feet/second .
Height with respect to time of Parachute.
Height with respect to time of mass.
Height above ground.
time
0
0.5
1
1.5
2
2.5
3
3.5
4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

The weight of the load in lbs = 10000 .
The stretch in lbs per foot applied to the bungee = 15 .
The diameter of the parachute in feet = 24 .
The release height is 3227.27 feet off of the ground.
Warning, bungee must be released immediately upon ground contact.
Landing velocity feet/sec if bungee had not been deployed 115.23.
Landing velocity in feet/sec with bungee deployment 4.23 .


The release height is
3236
feet.
The landing velocity will be
0
feet/second .
Height with respect to time of Parachute.
Height with respect to time of mass.
Height above ground.
time
3500
3000
2500
2000
1500
1000
500
0
0
5
10
15
20
25