Free Falling Objects and Reynolds Numbers

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Abstract

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- This presentation will discuss the underlying physics of the forces acting on a free-falling object, and the drag forces acting on those objects as well as Reynolds Numbers.
- From this knowlege, we will construct a differential equation to more properly explain the motion of falling objects of differing Reynolds Numbers and attempt to explain the vast error associated with neglecting the effects of said reynolds numbers.

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- Where m corresponds to the mass, a corresponds to the acceleration, and F equates to the sum of all forces acting on the object.
- We shall also make use of the following relationship: $a = \frac{dv}{dt}$

Basic Physics

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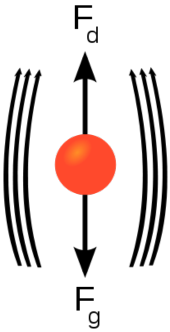
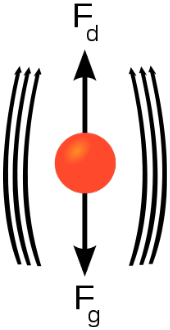
- $F_g + F_d = ma$



Basic Physics

- Note that F_g corresponds to the force of gravity, a negative number, and F_d corresponds to the drag force from air resistance, a positive number.



Classical Drag

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- Knowing this, we may modify our first equation to the form $-mg - kv = ma$
- To prepare our equation as a differential equation, it will change to $-mg - kv = m\frac{dv}{dt}$
- Though, as we will see, the drag force on an object is not always a mere linear relationship. To help us understand what may cause this, we will look to Reynolds Numbers.

Real Drag: Reynolds Numbers

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- Traditionally, mathematicians and engineers have approximated the drag force on any object to be $F_d = kv$ for all ranges of velocity. Unfortunately, this turns out to only be true for objects with a Reynolds number less than 1.
- For any Reynolds number greater than 1, the velocity dependence on the drag force will become exponential rather than linear.

Real Drag: Reynolds Numbers

- The Reynolds Number equation:

$$R = \frac{PDV}{u}$$
$$P = \text{Fluid Density}$$
$$D = \text{Characteristic Object Length}$$
$$V = \text{Fluid Velocity}$$
$$u = \text{Fluid Viscosity}$$

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- In other words, for varying Reynolds numbers, the force of drag equation is written as such:

$$F_d = kv^n$$

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- For example, while a Reynolds number less than 1 keeps a linear relationship, a Reynolds number above 10^3 may give a quadratic relationship.
- Though the true exponent n for an object will usually require a wind-tunnel to calculate, we may still use the knowlege that the drag force for objects is frequently not linear.
- Realistic Reynolds numbers may range from a value of 1 for a dust particle moving through air to a value of more than 10^8 for a submarine sinking in water.

Modifying the Equation

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- Assuming $v(0) = 0$ for our equation, we now have a DE that we might be able to work with.

Solving for different values of n

- Now that our equation includes the affect of Reynolds Numbers, we may notice that the value of n will have a great impact on our ability to solve and analyze the problem at hand:

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$$-mg - kv^n = m\frac{dv}{dt}$$

Solving for different values of n

- While the equations for the first few values of n may be easily solved, many values of n may become very tedious, if not impossible to solve.

$$\frac{dv}{dt} + \frac{k}{m}v = -g$$
$$\frac{dv}{dt} + \frac{k}{m}v^2 = -g$$
$$\frac{dv}{dt} + \frac{k}{m}v^3 = -g$$
$$\frac{dv}{dt} + \frac{k}{m}v^4 = -g$$

Matlab's Assistance

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- In order to demonstrate the effect of differing values of n on the maximum speed of the free-falling object, otherwise known as the object's "Terminal Velocity", we shall analyze Matlab-solved graphs of objects with static m and g values and differing n values.
- In this way, we may show the vast error when applying the linear equation to an object with a very large reynolds number.

Matlab Graph: n=1

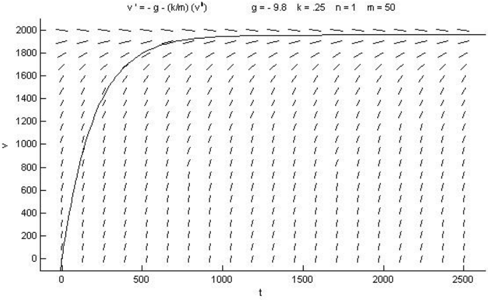

Figure: $n = 1$, Terminal Velocity Approx $= 1960$

Matlab Graph: n=2

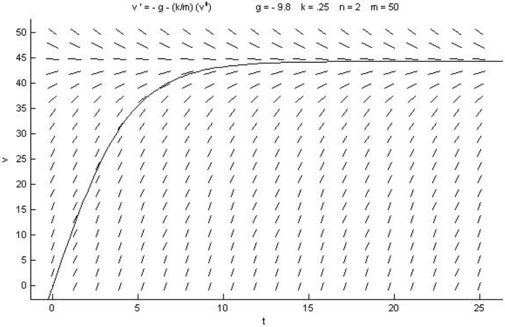

Figure: $n = 2$, Terminal Velocity Approx $= 44$

Matlab Graph: n=3

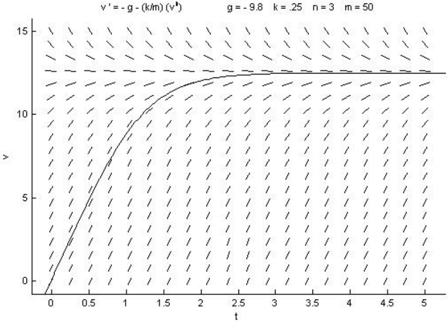

Figure: $n = 3$, Terminal Velocity Approx $= 12.5$

Matlab Graph: n=4

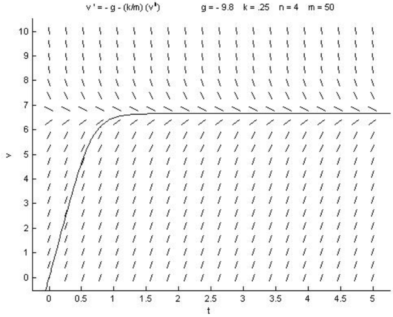

Figure: $n = 4$, Terminal Velocity Approx $= 6.6$

Analysis

- By calculating the terminal velocity of our generic equation, we find that the maximum velocity of an object is subject to being the root of its n value:

$$
\begin{aligned}
v_n &= \sqrt[n]{-\frac{mg}{k}} \\
v_1 &= -\frac{mg}{k} \\
v_2 &= \sqrt[2]{-\frac{mg}{k}}
\end{aligned}
$$

Analysis

- From the last few slides, anyone can clearly see the vast differences in the maximum terminal velocities of objects of differing reynolds numbers.

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- From the last few slides, anyone can clearly see the vast differences in the maximum terminal velocities of objects of differing reynolds numbers.
- We should note the real-world importance in recognizing these differences for applications in aircrafts, ships, cars, buildings... and just about everything that we build with aerodynamic and structural efficiency in mind.

Conclusion

- While this model is indeed superior to others, it still does not in fact truly represent the sum of all physics and knowledge. The model does not take into account various non-static factors that may be found within the variable k, and it assumes homogenous fluid density and viscosity and so forth.

Conclusion

- While this model is indeed superior to others, it still does not in fact truly represent the sum of all physics and knowledge. The model does not take into account various non-static factors that may be found within the variable k, and it assumes homogenous fluid density and viscosity and so forth.
- In conclusion, it is truly important for us to understand the vast errors that may be made by blindly sticking with a simplifying assumption to a system, yet we should be thankful that we are still able to achieve accurate solutions for a fair amount of cases that the assumption is made upon.

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