Aerial Bombing Trajectory

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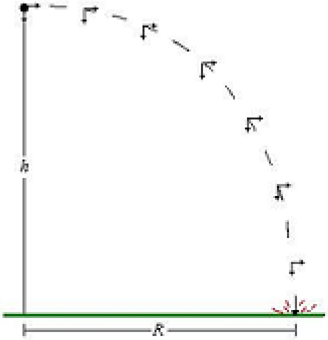
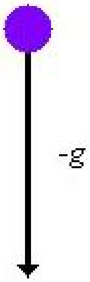


Vertical Motion

- Acceleration

$$m\frac{d^2y}{dt^2} = m(-g)$$
$$\frac{d^2y}{dt^2} = -g$$
$$a_y = -g$$

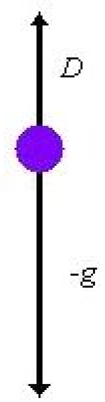


Vertical Motion

- Velocity

$$\frac{dy}{dt} = \int (-g)dt$$

$$\frac{dy}{dt} = -gt + C_1$$



Vertical Motion

- Initial Velocity

If $dy/dt = v_y$ and $C_1 = 0$ the velocity equation is

$$\frac{dy}{dt} = -gt + C_1$$
$$\frac{dy}{dt} = -gt + 0$$
$$v_y = -gt.$$

Vertical Motion

- Position

$$y = -\int gt\,dt$$

$$y = -g\int t\,dt$$

$$y = -\frac{1}{2}gt^2 + h$$

Horizontal Motion

- Acceleration

$$\frac{d^2x}{dt^2} = a_x$$

Since the object is deployed from a plane with a constant airspeed the falling object has no forward acceleration.
Thus

$$a_x = 0.$$

Horizontal Motion

- Velocity

The planes airspeed s is transfered to the falling object. Therefore

$$\frac{dx}{dt} = \int dt$$
$$\frac{dx}{dt} = v$$
$$v_x = v.$$

Horizontal Motion

- Position

$$x = \int s dt$$

$$x = st$$

Resistance

- Drag Equation

The drag R depends on the following conditions; air density ρ, velocity v, air's viscosity and compressibility D, and the cross section area of the object A.

$$R = \frac{1}{2} D \rho A v^2$$

Resistance

- Terminal Velocity Equation

Terminal Velocity occurs when the gravitational force is balanced by the resistive force (drag), and the net force is zero.

$$\frac{d^2y}{dt^2} = g - \left(\frac{D\rho A}{2m}\right) v^2$$

$$a = g - \left(\frac{D\rho A}{2m}\right) v^2$$

$$0 = g - \left(\frac{D\rho A}{2m}\right) v^2$$

$$v_T = \sqrt{\frac{2mg}{D\rho A}}$$

Differintial Equations

- Vertical Direction

Incorporating drag in the vertical motion will produce the final differential equation.
First begin with the generic equation

$$F = F_g + F_d$$

where

$$\begin{aligned} F &= ma \\ F_g &= mg \\ F_d &= \frac{1}{2} D\rho A v. \end{aligned}$$

Differintial Equations

- Vertical Direction

Which produces the equation

$$ma = mg - \frac{1}{2}D\rho Av$$
$$a = g - \frac{D\rho Av}{2m}$$

Differintial Equations

- Vertical Direction

Substituting k for $D\rho A/2m$

$$a = g - kv$$

$$a = \frac{dv}{dt} = g - kv$$

Differinial Equations

- Vertical Direction

Seperate variables

$$\begin{aligned} \frac{dv}{dt} &= g - kv \\ dv &= (g - kv)\,dt \\ \frac{dv}{g - kv} &= dt \end{aligned}$$

and integrate

$$\int \frac{dv}{g - kv} = \int dt.$$

Differintial Equations

- Vertical Direction

Integrating

$$\int \frac{dv}{g - kv} = \int dt$$

gives the equation

$$\frac{ln\left(g - kv\right)}{k} = t + C$$
$$ln\left(g - kv\right) = -k\left(t + C\right)$$

Differintial Equations

- Vertical Direction

Continued

$$e^{ln(g-kv)} = e^{-kt+kC}$$
$$g - kv = e^{-kt}e^{kC}$$

Substitute b for e^{kC} gives

$$g - kv = be^{-kt}$$
$$kv = g - be^{-kt}$$
$$v = \frac{g}{k} - \frac{b}{k}e^{-kt}.$$

Differintial Equations

- Vertical Direction

Setting $v = 0$ at $t = 0$ produces

$$\begin{aligned} v &= \frac{g}{k} - \frac{b}{k}e^{-kt} \\ 0 &= \frac{g}{k} - \frac{b}{k}e^{-k(0)} \\ 0 &= \frac{g}{k} - \frac{b}{k} \\ g &= b \end{aligned}$$

Differintial Equations

- Vertical Direction

$$v = \frac{g}{k} - \frac{g}{k}e^{-kt}$$
$$v = \frac{g}{k}\left(1 - e^{-kt}\right)$$
$$v = \frac{g}{k}\left(1 - e^{-kt}\right)$$

Differintial Equations

- Vertical Direction

Finding the displacement

$$
\begin{aligned}
v &= \frac{g}{k}\left(1 - e^{-kt}\right) \\
\frac{dy}{dt} &= \frac{g}{k}\left(1 - e^{-kt}\right) \\
\int dy &= \frac{g}{k}\int\left(1 - e^{-kt}\right)dt \\
y &= \frac{g}{k}\left(t + \frac{e^{-kt}}{k}\right) + C.
\end{aligned}
$$

Differintial Equations

- Vertical Direction

Initial condition $y = y_0$ at $t = 0$

$$y_0 = \frac{g}{k}\left(0 + \frac{e^0}{k}\right) + C$$

$$y_0 = \frac{g}{k}\left(\frac{1}{k}\right) + C$$

$$y_0 = \frac{g}{k^2} + C$$

$$C = y_0 - \frac{g}{k^2}$$

Differintial Equations

- Vertical Direction

Plugging in the C value will give the final equation for displament

$$y = \frac{g}{k}\left(t + \frac{e^{-kt}}{k}\right) + y_0 - \frac{g}{k^2}$$
$$y = \frac{g}{k}t + \frac{g}{k^2}e^{-kt} + y_0 - \frac{g}{k^2}$$
$$y = y_0 + \frac{g}{k}t - \frac{g}{k^2}\left(1 - e^{-kt}\right).$$

Graphing the Trajectories